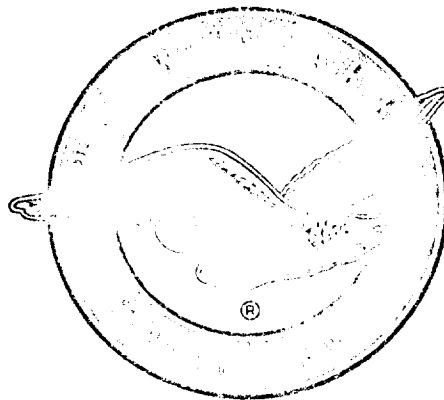


DIGITAL COMPUTER PROGRAMS FOR ROCKET NOZZLE DESIGN AND ANALYSIS

VOLUME IV SINGLE EXPANSION PLUG NOZZLE DESIGN

Prepared under Contract NAS9-2487
for NASA Manned Spacecraft Center



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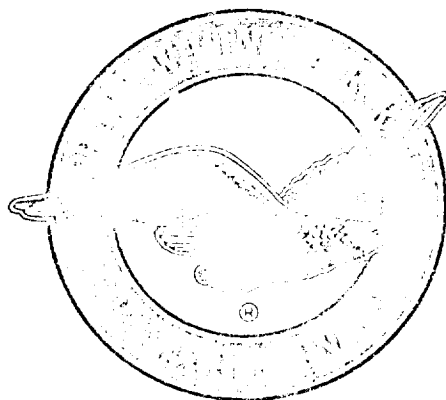
FLORIDA RESEARCH & DEVELOPMENT CENTER

UNITED STATES GOVERNMENT

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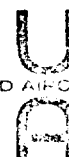
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FLORIDA RESEARCH & DEVELOPMENT CENTER

FOREWORD

This manual provides the necessary background for successful operation of the Single Expansion Plug Nozzle Design computer program. The manual was prepared under Contract NAS9-2487, Digital Computer Programs for Rocket Nozzle Design and Analysis, with the NASA Manned Spacecraft Center, Houston, Texas, and is the fourth of seven volumes specified in Part I of the contract.

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ABSTRACT

The necessary information for successful operation of the Single Expansion Plug Nozzle Design computer program is presented in this manual. Design criteria for the construction of a perfect plug nozzle and the order of calculations of the computer program is given with a discussion and flow diagram of each subroutine. The input required by the program is described and a sample output given.

No attempt is made to derive the equations used by the program. A general derivation of the basic equations, along with applications, is given in Volume I of this report.

SECTION I INTRODUCTION

The Single Expansion Plug Nozzle Design computer program constructs the supersonic nozzle contour of a perfect plug nozzle and calculates the corresponding performance. The method of characteristics for steady, supersonic potential flow is used in constructing the flow field. Thermodynamic properties can be based on either an ideal gas, where the gas properties of the combustion products are known, or approximated by a perfect gas.

Design criteria and a detailed description of the order of calculations is presented herein. Each of the subroutines used in the program is discussed and flow diagrams are given for clarification. The input and output formats for the program are included with recommended procedures to take in the event of unsuccessful runs.

Construction of the contour for a perfect plug nozzle requires that the throat flow be expanded to a uniform flow at the exit. However, the length and weight of such a nozzle may be excessive for use in any vehicle. Since a thrust loss of about 1% is obtained when the perfect contour is truncated by 90%, a significant reduction in weight can be obtained with a very small reduction in performance by simply truncating the perfect plug nozzle contour.

SECTION II TECHNICAL DESCRIPTION

In the design of a supersonic plug nozzle the throat flow is expanded to an axial direction in a manner such that uniform, shock free flow is obtained at the exit of the nozzle. The gas model and exit Mach number of the nozzle are the only parameters needed to construct the perfect nozzle contour when the location of the expansion point on the end of the afterbody section is specified. The method of characteristics for two-dimensional or axisymmetric potential flow is used to construct the necessary flow field.

The main program of the deck is used to control the order of calculations, while calculations such as determining the fluid properties at an interior point of the flow field or performance parameters along the constructed nozzle contour are made in subroutines that are "called" by the main program or other subroutines. The functions of the main program and calculations made therein are described in Paragraph B, and descriptions of the individual subroutines are given in Paragraph C.

A. DESIGN CRITERIA

A typical perfect single expansion plug nozzle contour is shown in figure 1. Expansion occurs along the entire supersonic nozzle contour, C_1C_2 , with the point of expansion, P_1 , located at the end of the afterbody section. Since all parameters are nondimensionalized within the program, the point P_1 is considered to be at location (0,1).

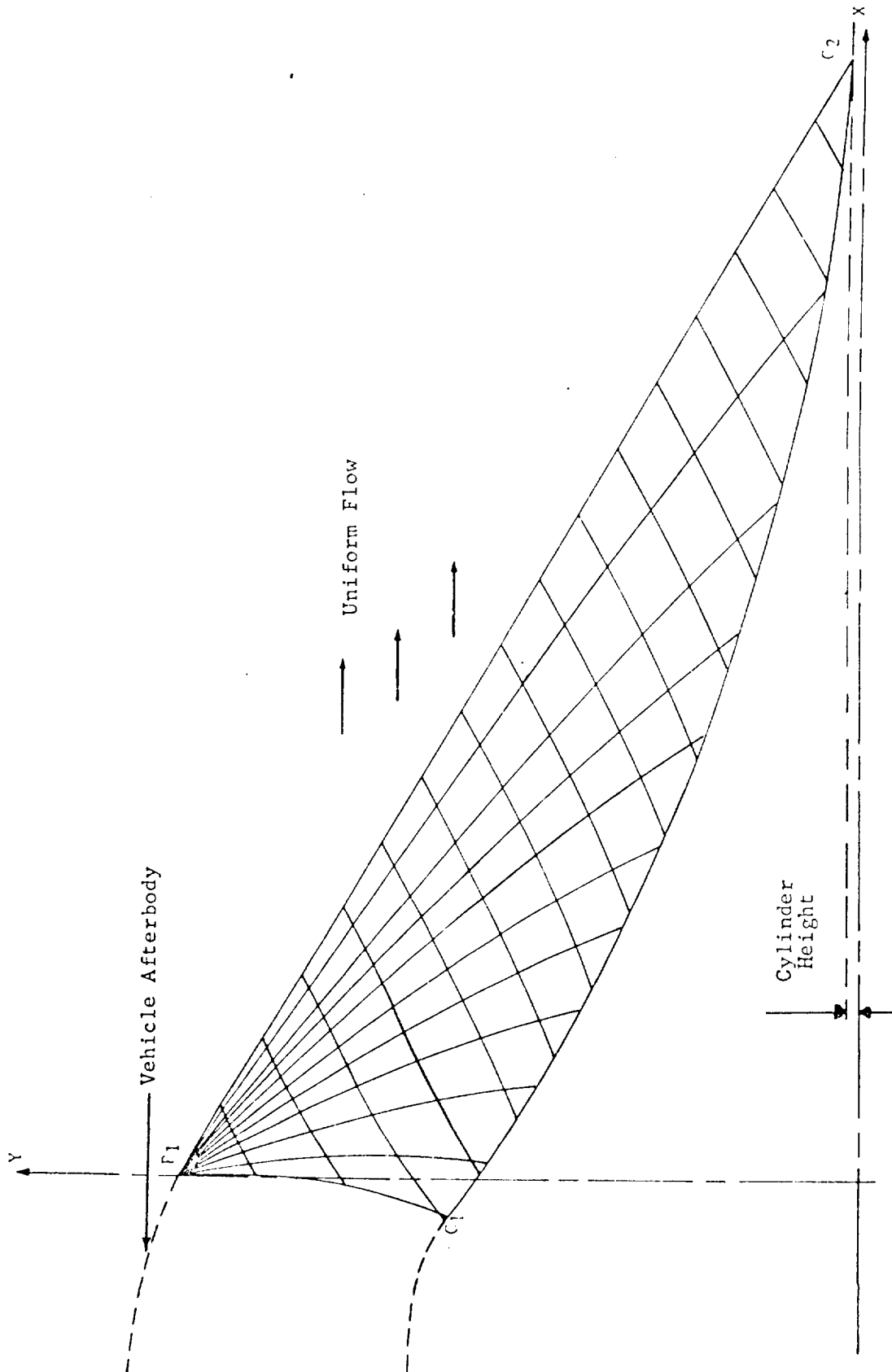


Figure 1. Typical Perfect Single Expansion Plug Nozzle Contour

1. Gas Model

In many cases, the thermodynamic properties of the nozzle exhaust may be approximated by assuming a perfect gas; in which case only the specific heat ratio, γ , must be input and the thermodynamic properties are calculated using perfect gas relationships. For this condition, the local velocity is nondimensionalized with respect to the maximum velocity (V_{\max}), and the critical velocity ratio (i.e., where $M = 1$) and density ratio at the throat are

$$W_{\text{sonic}} \equiv \frac{V_{\text{sonic}}}{V_{\max}} = \sqrt{\frac{\gamma-1}{\gamma+1}}$$

and

$$\frac{\rho_{\text{sonic}}}{\rho_o} = \left[1 - W_{\text{sonic}}^2 \right]^{\frac{1}{\gamma-1}}$$

The flow field for an ideal gas (usually equilibrium or frozen flow) may be calculated by specifying the thermodynamic properties (in tabular form) as a function of specific impulse. These properties may be obtained from conventional one-dimensional combustion programs*, and consist of pressure, density, local frozen sound speed (optional), and specific impulse. The SONICP Subroutine "beam fits" these properties as a function of specific impulse and determines the sonic velocity and density at the throat. The local velocities are then nondimensionalized with respect to the sonic velocity and the thermodynamic properties are beam fit as a function of the velocity ratio.

*Zelenik, F. S., and S. Gordon, NASA TN D-1454, "A General IBM 704 or 7090 Computer Program for Computation of Chemical Equilibrium Compositions, Rocket Performance, and Chapman-Jouquet Detonations".

2. Design Mach Number

Although parameters such as the nozzle area ratio, design pressure ratio, or the mass flow rate through the nozzle may be used to describe the exit conditions, this program uses the Mach number at the exit of the nozzle. When used in conjunction with the gas model and the location of the end of the vehicle afterbody, this parameter uniquely defines the supersonic plug nozzle contour. The input parameter used to describe the design Mach number is EM.

B. FLOW FIELD CONSTRUCTION

The first function of the main program is to call the INPUT Subroutine, which initializes parameters and reads in the input data. After calculating the velocity ratio at the exit and throat of the nozzle, the mass flow through the nozzle is calculated from $[\rho V A]_E$. The exit Mach line is then constructed using segments of equal length with uniform properties along the line and the accumulated mass flow from P_1 (figure 1) calculated at each point.

The exit Mach line is not extended to the axis of symmetry, since, for axisymmetric flow, numerical difficulties arise due to an indeterminate term at $Y = 0$ in the compatibility equations. Therefore, a cylinder of very small radius is placed around the axis (figure 1). This cylinder permits the solution to be approximated quite accurately, while eliminating the instabilities that may occur when trying to determine the limiting value as $Y \rightarrow 0$. Although this procedure is not required for two-dimensional flow, it is usually used to eliminate problems that may arise when calculating succeeding down Mach lines.

The throat area is a function of the mass flow, and is calculated by

$$A^* = \frac{\dot{m}}{\rho_{\text{sonic}} W_{\text{sonic}}}$$

To begin the flow field construction, the flow at the end of the vehicle afterbody is compressed through a small increment of velocity using the procedure in EXPAND Subroutine. From the expansion point, a down Mach line from B_1 to B_{n-1} (figure 2) is constructed by calculating the interior point intersections with up Mach lines from the points on the exit Mach line. The accumulated mass flow at each interior point is obtained by summing the mass flow across each up Mach line to the exit Mach line with the accumulated mass flow at the corresponding point on the exit Mach line (i.e., $A_1 A_3 B_3$). A point on the plug nozzle contour is then obtained by calculating the intersection of the streamline through A_n and the down Mach line $B_1 B_{n-1}$ (PLUGPT Subroutine).

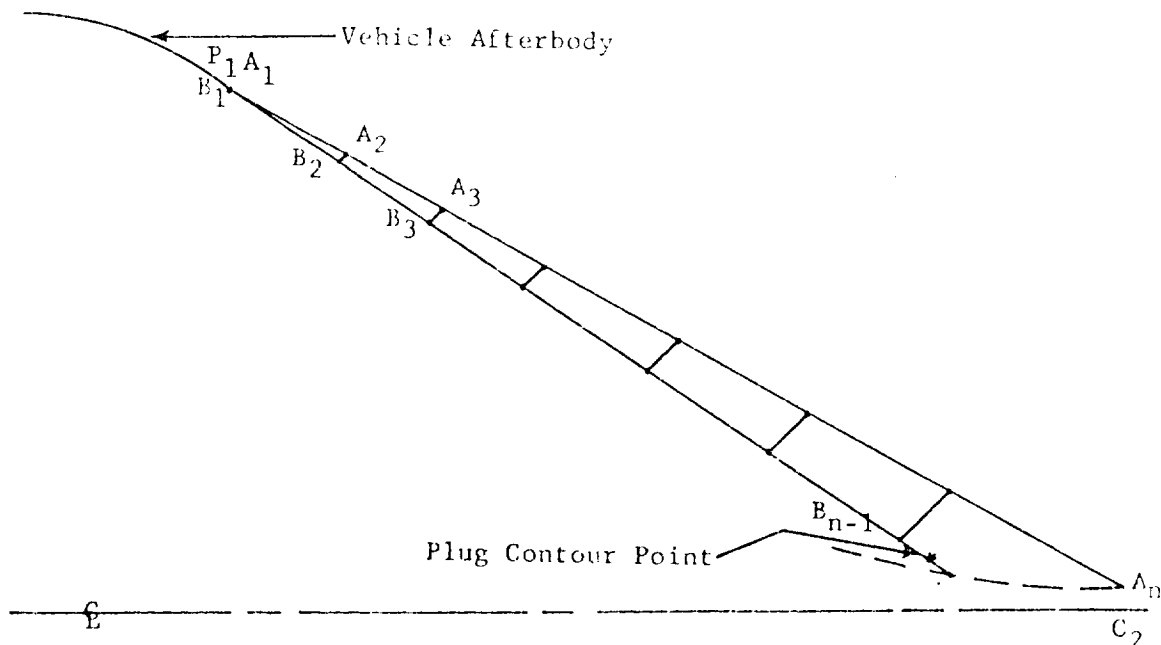


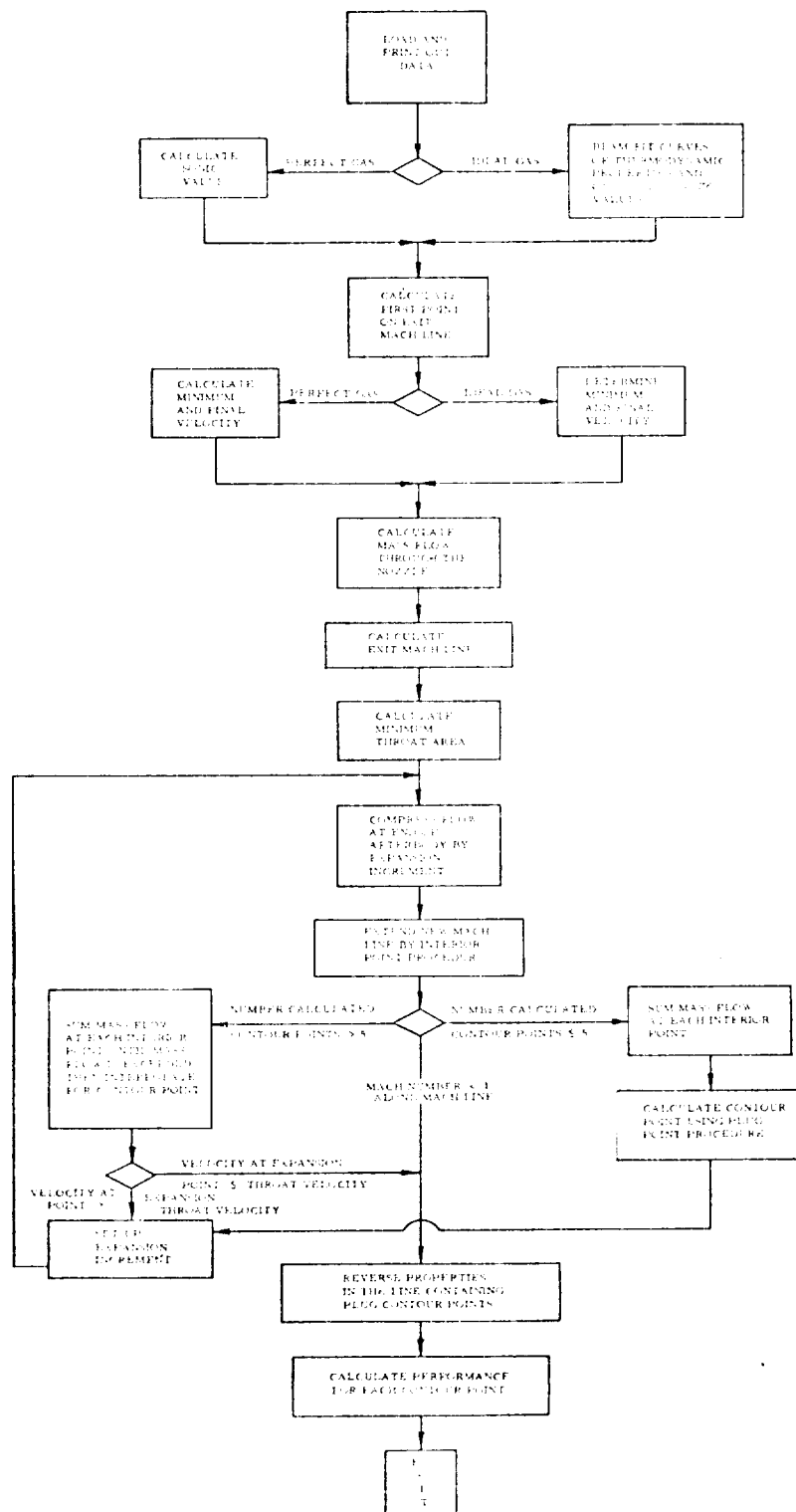
Figure 2

This procedure is used in constructing the first five down Mach lines, after which, the interior point intersections are calculated until the accumulated mass flow integrated along the down Mach line exceeds the total mass flow through the nozzle. The contour point is then determined by linear interpolation. The construction of new down Mach lines is continued until (1) the velocity at the compression (or expansion) point is equal to the throat velocity, (2) the flow becomes subsonic along the down Mach line, or (3) the Y coordinate along the contour exceeds the value of 1.

Having constructed the entire plug contour, the PERFO Subroutine is used to calculate and print the following performance parameters:

1. X/R
2. Y/R
3. TAN THETA (Contour Slope)
4. MACH NO.
5. P/PC (Local static pressure/chamber pressure)
6. GAMMA (Specific heat ratio)
7. AS/A^* (Local surface area/throat area)
8. CTG (Gross thrust coefficient)
9. CTN (Net thrust coefficient).

Single Expansion Plug Nozzle Design



C. SUBROUTINES

Since most of the subroutines are used many times in constructing a flow field or calculating performance, they are discussed individually in this section. The purpose of each subroutine, the equations used, and flow diagrams are given.

1. INTX Subroutine

Under certain conditions, the numerical solution of the characteristic system becomes difficult or impossible in determining the intersection of an up and a down Mach line. These conditions may occur if the slope of one or both of the Mach lines is extremely large or small. The problem can be eliminated by rotating the coordinate axis when solving the physical characteristics, and by modifying the axisymmetric term in the compatibility equations. The physical characteristic equations are invariant under this transformation. The INTX Subroutine performs the function of determining if rotation is needed, the form required for the axisymmetric term, and calls one of the following subroutines:

INT1 Subroutine - No rotation is used and the axisymmetric term of the compatibility equations for both up and down Mach lines use the differential dX .

INT2 Subroutine - The coordinate system is rotated and the axisymmetric term of the compatibility equations for both up and down Mach lines use the differential dX (for an up or down Mach line with very small slope).

INT3 Subroutine - The coordinate system is rotated and the axisymmetric term of the compatibility equations for both up and down Mach lines use the differential dY (for an up or down Mach line with very large slope).

INT4 Subroutine - The coordinate system is rotated and the axisymmetric term of the up Mach line uses dX and the down Mach line dY (for an up Mach line with very small slope combined with a down Mach line with very large slope).

INT5 Subroutine - The coordinate system is rotated and the axisymmetric term of the up Mach line uses dY and the down Mach line dX (for an up Mach line with very large slope combined with a down Mach line with very small slope).

The coordinates and flow conditions (W , $\tan \theta$, and $\tan \alpha$) at the points on an up and down Mach line must be stored into the variables $A(I)$ and $B(I)$, respectively.

```

graph TD
    ENTER((ENTER)) --> INIT[EPSILON = 0.5  
TEMP1 = A(1) * A(1)  
TEMP2 = 1 - A(1) * A(1)  
TEMP3 = B(1) * B(1)  
TEMP4 = 1 - B(1) * B(1)]
    INIT --> D1{TEMP1 >= 100.0}
    D1 -- "yes" --> INT1[CALL INT1  
T2 = 1]
    D1 -- "no" --> D2{TEMP1 <= 10000}
    D2 -- "yes" --> INT2[CALL INT2  
T2 = 1]
    D2 -- "no" --> D3{TEMP1 < EPSILON}
    D3 -- "yes" --> INT3[CALL INT3  
T2 = 1]
    D3 -- "no" --> D4{TEMP2 < EPSILON}
    D4 -- "yes" --> INT4[CALL INT4  
T2 = 1]
    D4 -- "no" --> D5{TEMP3 < EPSILON}
    D5 -- "yes" --> INT5[CALL INT5  
T2 = 1]
    D5 -- "no" --> D6{TEMP4 < EPSILON}
    D6 -- "yes" --> INT6[CALL INT6  
T2 = 1]
    D6 -- "no" --> INT6
    INT1 --> STOP[STOP]
    INT2 --> STOP
    INT3 --> STOP
    INT4 --> STOP
    INT5 --> STOP
    INT6 --> STOP
  
```


2. INT1 Subroutine

Given the coordinates and flow conditions (W , $\tan \theta$, and $\tan \alpha$) at two points in the flow field not on the same Mach line, the INT1 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point, and a down Mach line from the other. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the interior point, 3, (figure 3) will be stored in the variable C(I).

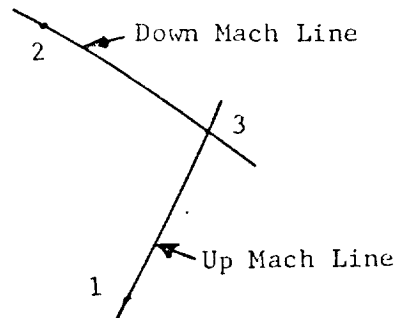


Figure 3

Written in finite difference form, the characteristic system is

$$(Y_3 - Y_1) = \left[\frac{\tan \theta + \tan \alpha}{1 - \tan \alpha \tan \theta} \right]_1 (X_3 - X_1) \quad (2.1)$$

and

$$(Y_3 - Y_2) = \left[\frac{\tan \theta - \tan \alpha}{1 + \tan \alpha \tan \theta} \right]_2 (X_3 - X_2),$$

$$(W_3 - W_1) \left[\frac{1}{W \tan \alpha} \right]_1 - (\tan \theta_3 - \tan \theta_1) \left[\frac{1}{1 + \tan^2 \theta} \right]_1 = c \left[\frac{\tan \alpha \tan \theta}{(1 - \tan \alpha \tan \theta) Y} \right]_1 (X_3 - X_1)$$

and (2.2)

$$(W_3 - W_2) \left[\frac{1}{W \tan \alpha} \right]_2 + (\tan \theta_3 - \tan \theta_2) \left[\frac{1}{1 + \tan^2 \theta} \right]_2 = c \left[\frac{\tan \alpha \tan \theta}{(1 + \tan \alpha \tan \theta) Y} \right]_2 (X_3 - X_2).$$

The subscripts 1 and 2 indicate that the quantities in brackets are evaluated at these points.

Solving equations (2.1) simultaneously, the coordinates at point 3

are:

$$X_3 = \frac{Y_1 - \left[\frac{\tan \theta + \tan \alpha}{1 - \tan \alpha \tan \theta} \right]_1 X_1 - Y_2 + \left[\frac{\tan \theta - \tan \alpha}{1 + \tan \alpha \tan \theta} \right]_2 X_2}{\left[\frac{\tan \theta - \tan \alpha}{1 + \tan \alpha \tan \theta} \right]_2 - \left[\frac{\tan \theta + \tan \alpha}{1 - \tan \alpha \tan \theta} \right]_1} \quad (2.3)$$

and

$$Y_3 = Y_2 + (X_3 - X_2) \left[\frac{\tan \theta - \tan \alpha}{1 + \tan \alpha \tan \theta} \right]_2.$$

Flow conditions W_3 and $\tan \theta_3$ are then obtained from the simultaneous solution of equations (2.2).

$$W_3 = \frac{K_1 + K_2}{\left[\frac{1}{W \tan \alpha} \right]_2 \div \left[\frac{1}{1 + \tan^2 \theta} \right]_2 + \left[\frac{1}{W \tan \alpha} \right]_1 \div \left[\frac{1}{1 + \tan^2 \theta} \right]_1}$$

and

$$\tan \theta_3 = K_2 + W_3 \left[\frac{1}{W \tan \alpha} \right]_1 \div \left[\frac{1}{1 + \tan^2 \theta} \right]_1; \quad (2.4)$$

where:

$$K_1 = \tan \theta_2 + W_2 \frac{\left[\frac{1}{W \tan \alpha} \right]_2}{\left[\frac{1}{1 + \tan^2 \theta} \right]_2} + \sigma (X_3 - X_2) \frac{\left[\frac{\tan \alpha \tan \theta}{(1 + \tan \alpha \tan \theta) Y} \right]_2}{\left[\frac{1}{1 + \tan^2 \theta} \right]_2}$$

and

$$K_2 = \tan \theta_1 - W_1 \frac{\left[\frac{1}{W \tan \alpha} \right]_1}{\left[\frac{1}{1 + \tan^2 \theta} \right]_1} + \sigma (X_1 - X_3) \frac{\left[\frac{\tan \alpha \tan \theta}{(1 - \tan \alpha \tan \theta) Y} \right]_1}{\left[\frac{1}{1 + \tan^2 \theta} \right]_1}$$

The tangent of the Mach angle ($\tan \alpha_3$), which is a function of W_3 , is determined by the procedure described in TAGAL Subroutine for an ideal gas or by the procedure in the PRFCT Subroutine for a perfect gas.

Since the evaluation of equations (2.3) and (2.4) gives first approximations to X_3 , Y_3 , $\tan \theta_3$, W_3 , and $\tan \alpha_3$, improved solutions are obtained by replacing the quantities in brackets with average values; that is,

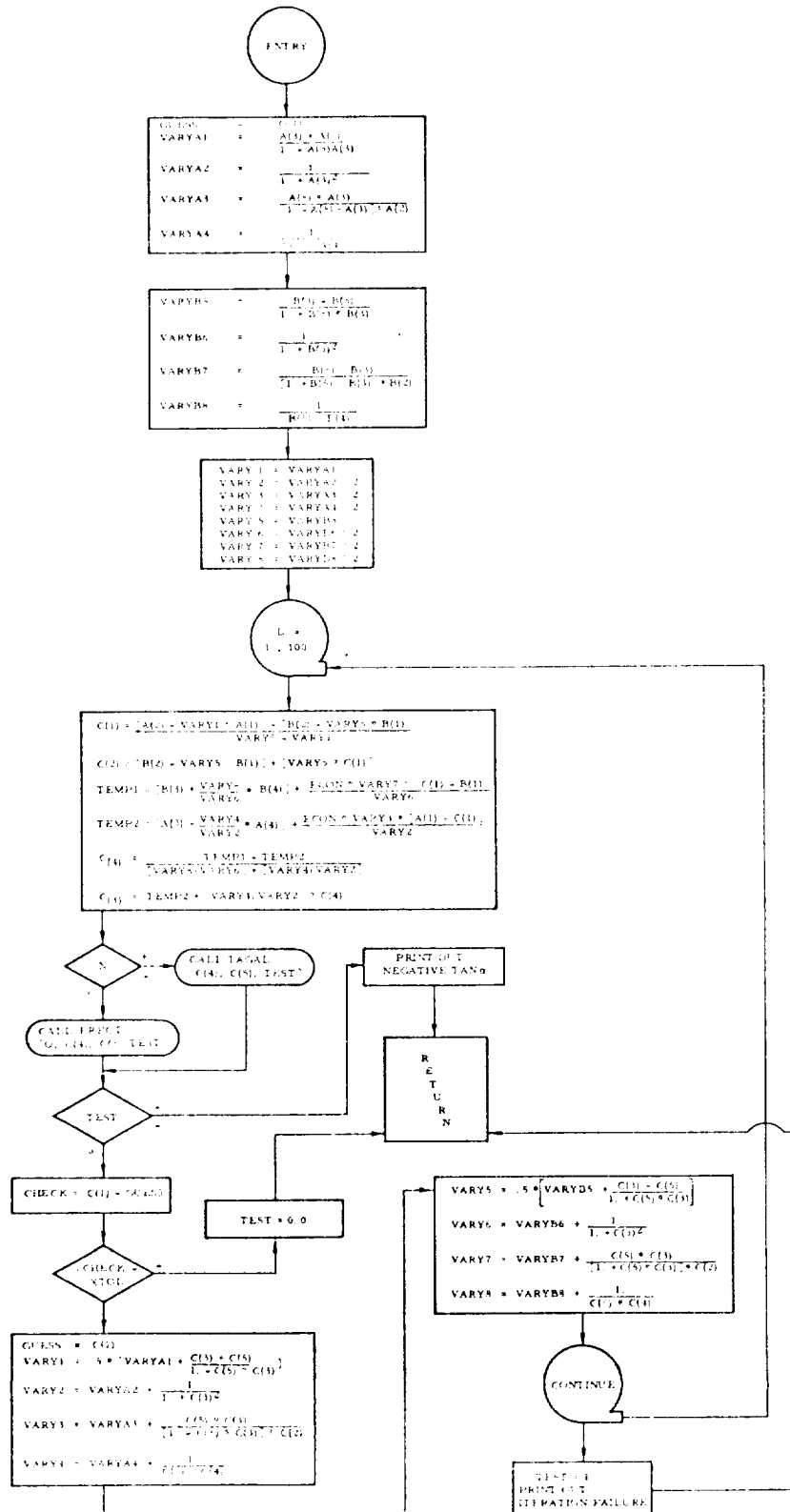
replace $\left[\frac{\tan \theta + \tan \alpha}{1 - \tan \alpha \tan \theta} \right]_1$ by

$$\left[\frac{\tan \theta + \tan \alpha}{1 - \tan \alpha \tan \theta} \right]_{1,3} = 1/2 \left\{ \left[\frac{\tan \theta + \tan \alpha}{1 - \tan \alpha \tan \theta} \right]_1 + \left[\frac{\tan \theta + \tan \alpha}{1 - \tan \alpha \tan \theta} \right]_3 \right\}$$

This procedure for obtaining the improved solutions is repeated until successive values of X_3 are within a specified tolerance.

$$\left| X_3^i - X_3^{i-1} \right| \leq XTOL$$

Subroutine INT 1



3. INT2 Subroutine

Given the coordinates and flow conditions (W , $\tan \theta$, and $\tan \alpha$) at two points in the flow field not on the same Mach line, the INT2 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of either Mach line is very small. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables $A(I)$ and $B(I)$, respectively. The corresponding properties at the intersection point, 3, (figure 4) will be stored in the variable $C(I)$.

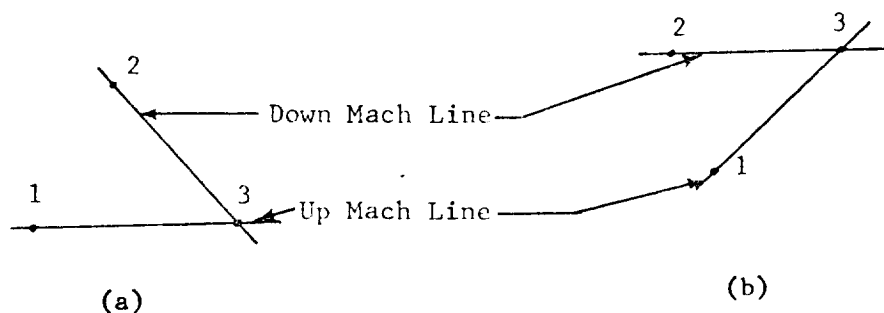


Figure 4

The characteristic system is the same as for the INT1 Subroutine (equations 2.1 and 2.2), except that the coordinate axes are rotated. The coordinate transformation is given by the following:

$$X' = X \cos \varphi + Y \sin \varphi$$

$$Y' = Y \cos \varphi - X \sin \varphi,$$

and

(3.1)

$$\tan \theta' = (\tan \theta - \tan \varphi) \div (1 + \tan \theta \tan \varphi)$$

where:

the prime indicates the rotated value and φ the angle of rotation.

Solving the physical characteristics (equation 2.1) simultaneously using the rotated values, the coordinates at point 3' are determined. The coordinate axes are then rotated back to their original position, and the flow conditions W_3 and $\tan \theta_3$ are obtained as in the INT1 Subroutine.

4. INT3 Subroutine

Given the coordinates and flow conditions (W , $\tan \theta$, and $\tan \alpha$) at two points in the flow field not on the same Mach line, the INT3 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of either Mach line is very large. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the intersection point, 3, (figure 5) will be stored in the variable C(I).

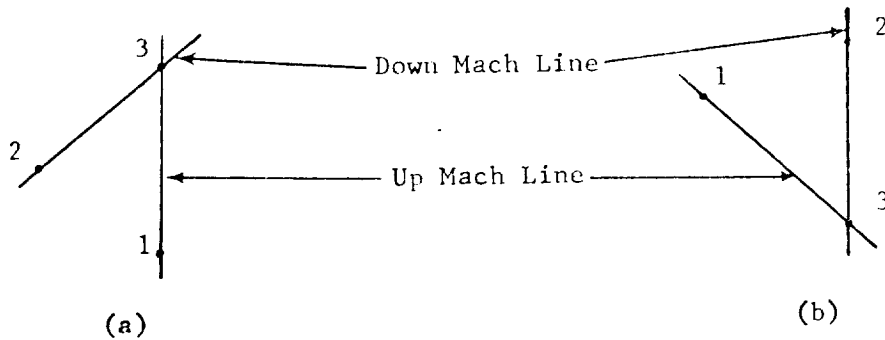


Figure 5

The characteristic system is the same as for the INT1 Subroutine (equations 2.1 and 2.2), except that the axisymmetric term of the compatibility equations have the form

$$\sigma \left[\frac{\tan \alpha \tan \theta}{(\tan \theta + \tan \alpha) Y} \right]_1 (Y_3 - Y_1) \quad (4.1)$$

for an up Mach line, and

$$\sigma \left[\frac{\tan \alpha \tan \theta}{(\tan \theta - \tan \alpha) Y} \right]_2 (Y_3 - Y_2) \quad (4.2)$$

for a down Mach line.

ENTER

```

C = F * S * V
VARYING = 1 - ((1 + A(S)) / (1 + A(S) * A(5))) * A(Z)
VARYING = 1 - ((A(S) * A(4)) / (1 + B(S)))
VARYING = 2 * ((S * B(S)) / (1 + B(S) * B(S) + 2 * Z))
VARYING = 1 - ((B(S) * P(R))

```

$$\begin{aligned} \text{COSP} &= \text{COS}(\text{DOSP}) \\ \text{SINF} &= \text{SINF}(\text{DOSP}) \\ \text{TANF} &= \text{TAN}(\text{DOSP}) \\ \text{XA} &= \text{A} \cdot \text{X} \\ \text{YA} &= \text{A} \cdot \text{Y} \\ \text{XB} &= \text{B} \cdot \text{X} \\ \text{YB} &= \text{B} \cdot \text{Y} \end{aligned}$$

```

A1 := M1 - C1 * S1 + A1 * C1 * S1
IA := C1 * T1 + C1 * S1 + A1 * C1 * T1 * S1
P11 := B1 * C1 * S1 + C1 * S1 * S1
B1 := B1 * C1 * C1 + A1 * C1 * S1
TR := C1 * T1 + T1 * S1 + B1 * C1 * T1 * S1
VARP1 := IA * A1 + A1 * T1 - T1 * A1 * C1
VARTR := TR * P11 - P11 * (1 + TR * B1)
VAR1 := VAR1
VAR2 := C1 * VAR2
VAR3 := C1 * VAR3
VAR4 := C1 * VAR4
VAR5 := VAR5
VAR6 := C1 * VAR6
VAR7 := C1 * VAR7
VAR8 := C1 * VAR8

```

$$\left(\begin{array}{c} 1 \\ 1 \end{array} \right) = 1, 100$$

```

C(1)=C(1)+VARY*(1-C(1)+C(2)+VARY**2)/(1+VARY+VARY**2)
C(2)=C(2)+VARY*(1-C(1)+C(2)+VARY**2)/(1+VARY+VARY**2)
N=C(1)
C(1)=C(1)+C(2)+VARY**2
C(2)=C(2)+C(1)+C(2)+VARY**2
TEMP1=C(1)+C(2)+VARY**2
TEMP2=C(1)+VARY*(1-C(1)+C(2)+VARY**2)/(1+VARY+VARY**2)
C(1)=TEMP1
C(2)=TEMP2
VARY=VARY*(1+VARY+VARY**2)

```

CALIF. TAX AC.
C74, *C8*, *E

CALL 800-2
10, 000, 000, 000

25 T

True

$$U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```

C1:=S1*(1-
TC:=TC+TAN(1-TC)*TAN(1-
VAR1:=1-1/2*VAR1+1/2*(S1-1-
VAR2:=VAR2+1/2*(C1-TC)
VAR3:=VAR3+(S1-TC)*TC-1/2*(C1-TC)
VAR4:=VAR4+(1-TC)*TC
VAR5:=1/2*(1-TC)*VAR1+1/2*(S1-1-
VAR6:=VAR6+1/2*(C1-TC)
VAR7:=VAR7+(S1-TC)*TC-1/2*(C1-TC)
VAR8:=VAR8+1/2*(C1-TC)
VAR9:=VAR9+(S1-TC)*TC-1/2*(C1-TC)

```

CONFIDENTIAL

```

TEST 41
PRINT MORE THAN 10 ITER-
ATIONS IN INTERSUBJECTIVE

```

T(1) = 0
 A(1) = NA
 A(2) = YA
 B(1) = NA
 B(2) = YB

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1

Because numerical solution of the physical characteristics is extremely difficult when the slope of a Mach line is very large, the coordinate axes must be rotated. The coordinate transformation and the calculation of flow conditions are the same as presented in the INT2 Subroutine.

5. INT4 Subroutine

Given the coordinates and flow conditions (W , $\tan \theta$, and $\tan \alpha$) at two points in the flow field not on the same Mach line, the INT4 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of the up Mach line is very small and the slope of the down Mach line very large. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables $A(I)$ and $B(I)$, respectively. The corresponding properties at the intersection point, 3, (figure 6) will be stored in the variable $C(I)$.

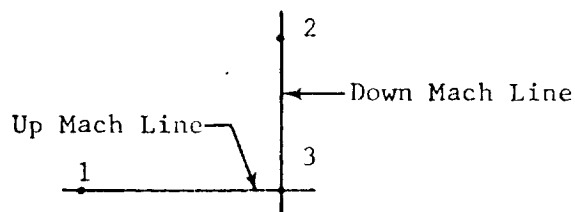


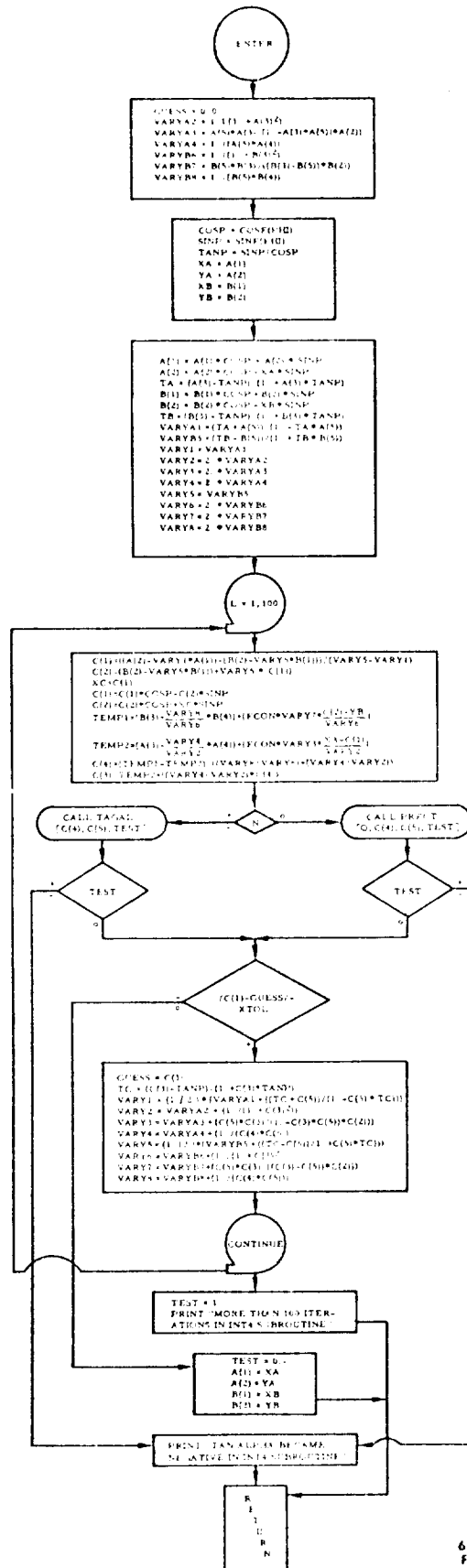
Figure 6

The characteristic system is the same as in INT1 Subroutine (equations 2.1 and 2.2), except that the axisymmetric term of the compatibility equation for the down Mach line is

$$\sigma \left[\frac{\tan \alpha \tan \theta}{(\tan \theta - \tan \alpha) Y} \right]_2 (Y_3 - Y_2). \quad (5.1)$$

Because numerical solution of the physical characteristics is extremely difficult when the slope of a Mach line is large, the coordinate axes must be rotated. The coordinate transformation and the calculation of flow conditions are the same as presented in the INT2 Subroutine.

Subroutine INT4

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6. INT5 Subroutine

Given the coordinates and flow conditions (W , $\tan \theta$, and $\tan \alpha$) at two points in the flow field not on the same Mach line, the INT5 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of the up Mach line is very large and the slope of the down Mach line very small. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the intersection point, 3, (figure 7) will be stored in the variable C(I).

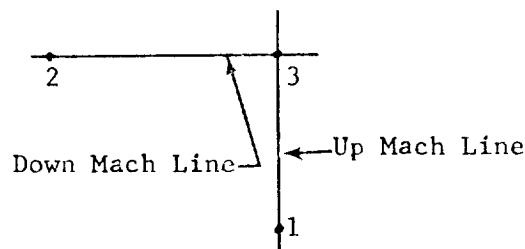


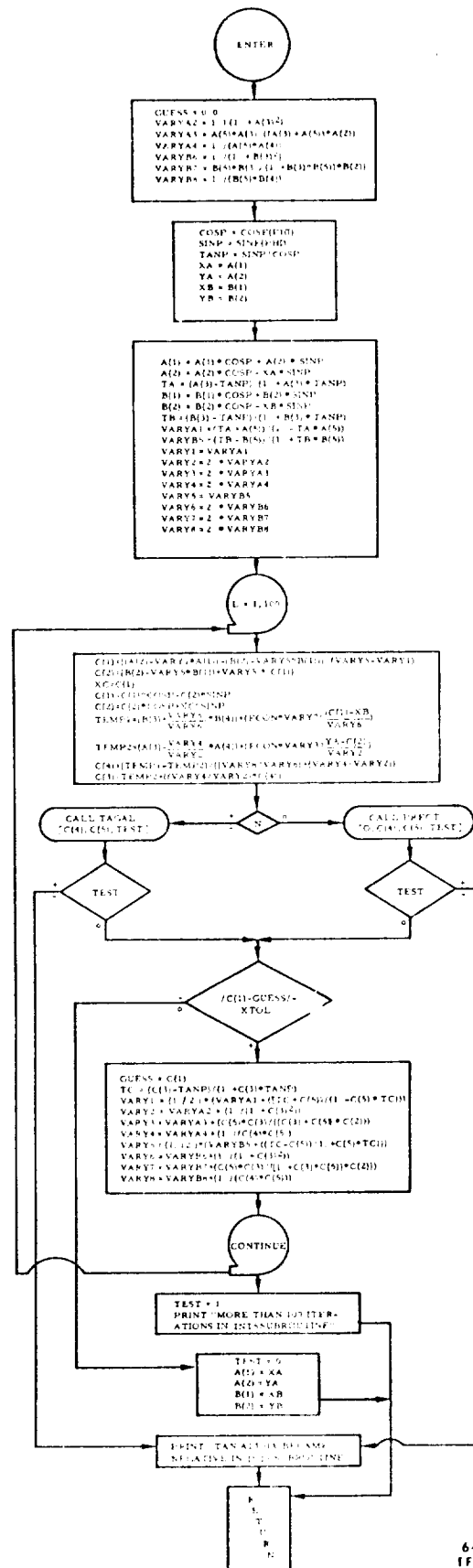
Figure 7

The characteristic system is the same for the INT1 Subroutine (equations 2.1 and 2.2), except that the axisymmetric term of the compatibility equation for the up Mach line is

$$\sigma \left[\frac{\tan \alpha \tan \theta}{(\tan \theta + \tan \alpha)Y} \right]_1 (Y_3 - Y_1). \quad (6.1)$$

Because numerical solution of the physical characteristics is extremely difficult when the slope of a Mach line is large, the coordinate axes must be rotated. The coordinate transformation and the calculation of flow conditions are the same as presented in the INT2 Subroutine.

Subroutine INT5



7. EXPAND Subroutine

The EXPAND Subroutine calculates the flow properties at a point after the flow has been compressed by an increment in the velocity ratio (figure 8). The slope, $\tan \theta_C$, of the compressed velocity ratio, W_C , is found

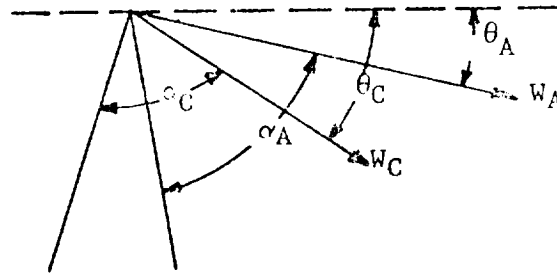


Figure 8

by integrating

$$\frac{d(\tan \theta)}{dW} = \frac{1 + \tan^2 \theta}{W \tan \alpha} = f(W, \tan \theta). \quad (7.1)$$

Using the method of Runge-Kutta over ten intervals,

let

$$h = \frac{W_C - W_A}{10}$$

$$W_1 = W_A$$

and

$$\tan \theta_1 = \tan \theta_A$$

Solve equations 7.2 to 7.8 as $j = 1, 10$.

$$K_1 = f(W_j, \tan \theta_j)h \quad (7.2)$$

$$K_2 = f(W_j + h/2, \tan \theta_j + K_1/2)h \quad (7.3)$$

$$K_3 = f(W_j + h/2, \tan \theta_j + K_2/2)h \quad (7.4)$$

$$K_4 = f(W_j + h, \tan \theta_j + K_3)h \quad (7.5)$$

$$\Delta(\tan \theta) = 1/6(K_1 + 2K_2 + 2K_3 + K_4) \quad (7.6)$$

$$\tan \theta_{j+1} = \tan \theta_j + \Delta(\tan \theta) \quad (7.7)$$

$$W_{j+1} = W_j + h \quad (7.8)$$

The properties corresponding to W_C are

$$W_C = W_{11}$$

$$\tan \alpha_C = f(W_C)$$

$$\tan \theta_C = \tan \theta_{11},$$

and are stored in the variable C(I). Since the compression occurs about a sharp corner, the X and Y coordinates remain unchanged.

8. PLUGPT Subroutine

The PLUGPT Subroutine is used to determine contour points by calculating the intersections of a down Mach line with the streamline corresponding to the total mass flow through the nozzle.

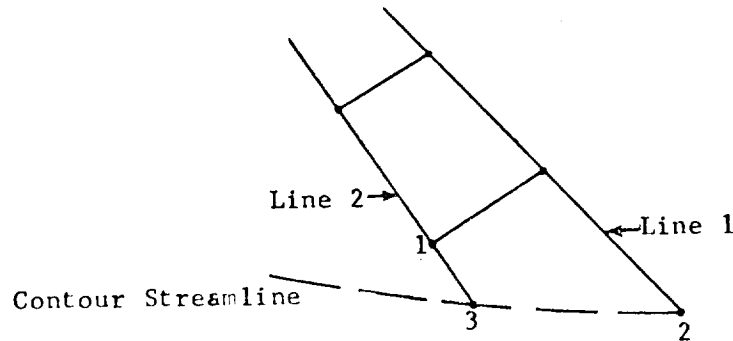


Figure 9

Before entering this subroutine the total mass flow must be stored in TMFLOW; the mass flow between the expansion point and point 1 must be in AMFLOW; and the flow conditions at points 1 and 2 must be in B(I) and BL(NUM, I), respectively. The calculated values at point 3 are stored in C(I).

The necessary first guess of $\tan \theta_3$ is the slope of the streamline at the last calculated point on the contour.

By solving simultaneously the physical characteristic equation and the equation of the streamline, the coordinates at point 3 are calculated by

$$X_3 = \frac{Y_1 - Y_2 + X_2 [\tan \theta]_2 - X_1 \left[\frac{\tan \theta - \tan \alpha}{1 + \tan \alpha \tan \theta} \right]_1}{[\tan \theta]_2 - \left[\frac{\tan \theta - \tan \alpha}{1 + \tan \alpha \tan \theta} \right]_1}, \quad (8.1)$$

and

$$Y_3 = Y_2 + (X_3 - X_2) [\tan \theta]_2. \quad (8.2)$$

The velocity at point 3 is

$$W_3 = W_1 - \frac{(\tan \theta_3 - \tan \theta_1) \left[\frac{1}{1 + \tan^2 \theta} \right]_1 - \sigma(Y_3 - Y_1) \left[\frac{\tan \alpha \tan \theta}{(\tan \theta - \tan \alpha) Y} \right]_1}{\left[\frac{1}{W \tan \alpha} \right]_1}, \quad (8.3)$$

and

$$\tan \alpha_3 = f(W_3). \quad (8.4)$$

Improved solutions to these equations are obtained by replacing the quantities in the brackets that are subscripted 1 with average values, and repeating the calculations until

$$\left| X_3^i - X_3^{i-1} \right| \leq 0.0000001,$$

and

$$\left| W_3^i - W_3^{i-1} \right| \leq 0.0000001.$$

Obtaining a new value for $\tan \theta_3$ and replacing $[\tan \theta]_2$ with the average value, $[\tan \theta]_{2,3}$, the above procedure is repeated until the calculated mass flow between points 1 and 3 is equal to $\left| \text{TMFLOW} - \text{AMFLOW} \right|$ within the optional input tolerance, TOLMF.

[illegible]

9. PRFCT Subroutine

The PRFCT Subroutine is made up of four perfect gas relationships, which are a function of velocity ratio and a constant specific heat ratio. Depending on the value of the parameter (L), this subroutine calculates either $\tan \alpha$, ratio of static to total density, ratio of static to total pressure, or Mach number.

The following equations are evaluated by the PRFCT Subroutine.

$$\tan \alpha = \sqrt{\frac{1 - W^2}{W^2 \left(\frac{\gamma + 1}{\gamma - 1} \right) - 1}} \quad (L = 0)$$

$$\rho/\rho_0 = [1 - W^2]^{\frac{1}{\gamma - 1}} \quad (L = 1)$$

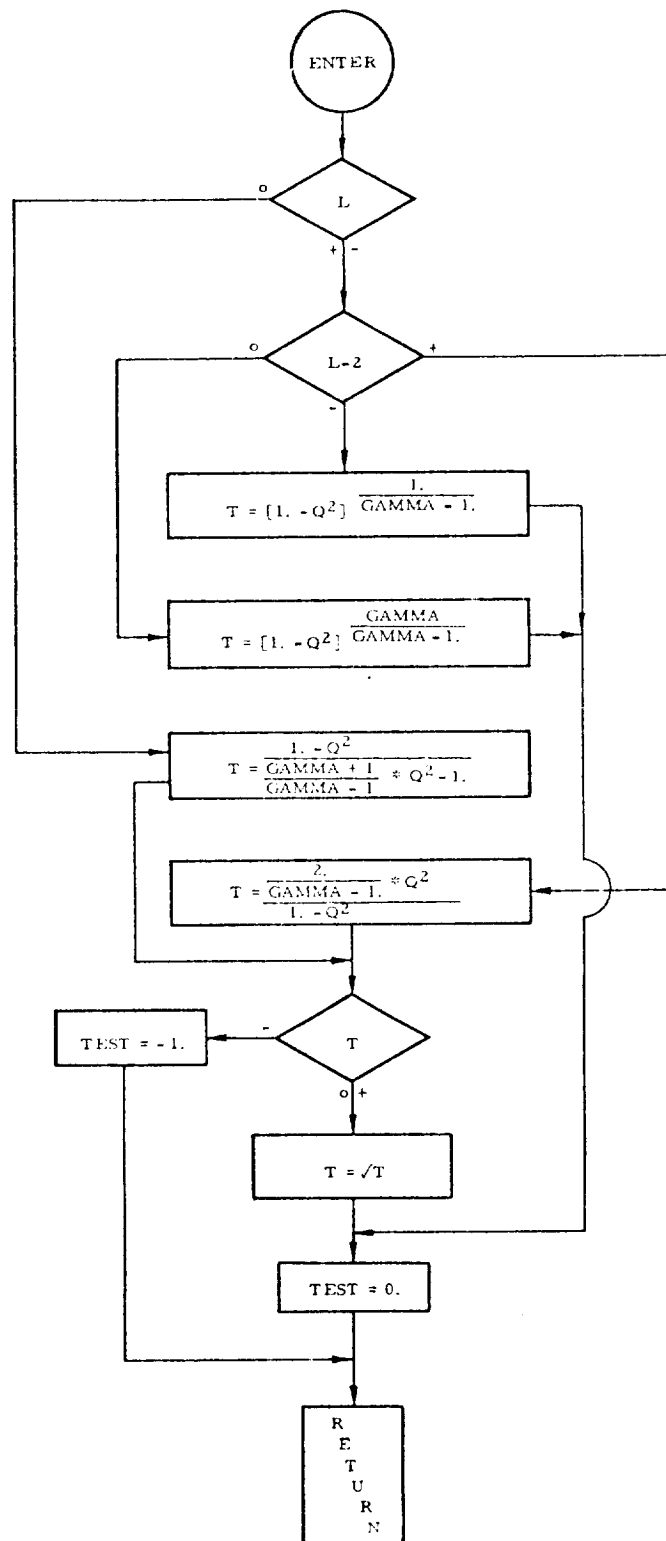
$$P/P_0 = [1 - W^2]^{\frac{\gamma}{\gamma - 1}} \quad (L = 2)$$

$$M = \sqrt{\frac{W^2 \left(\frac{2}{\gamma - 1} \right)}{1 - W^2}} \quad (L = 3)$$

The following is an explanation of the subroutine call list.

- L - Indicates parameter to be calculated
- Q - The known or input value of velocity ratio V/V_{\max}
- T - Variable that will contain the calculated value
- TEST - An error signal in case of a subsonic velocity.

Subroutine PRFCT



10. TAGAL Subroutine

For an ideal gas, the TAGAL Subroutine is used to calculate $\tan \alpha$ as a function of a known velocity ratio ($W = V/V_{\text{sonic}}$). For the option where the local frozen sound speeds from the table of gas properties are not used,

$$\tan \alpha = \sqrt{\frac{1}{V_{\text{sonic}}^2 W^2 \left(\frac{d\rho/dW}{dP/dW} \right) - 1}}$$

A beam fit evaluation of the gas properties is necessary to determine the values of $d\rho/dW$ and dP/dW .

If the local frozen sound speed option is used, then

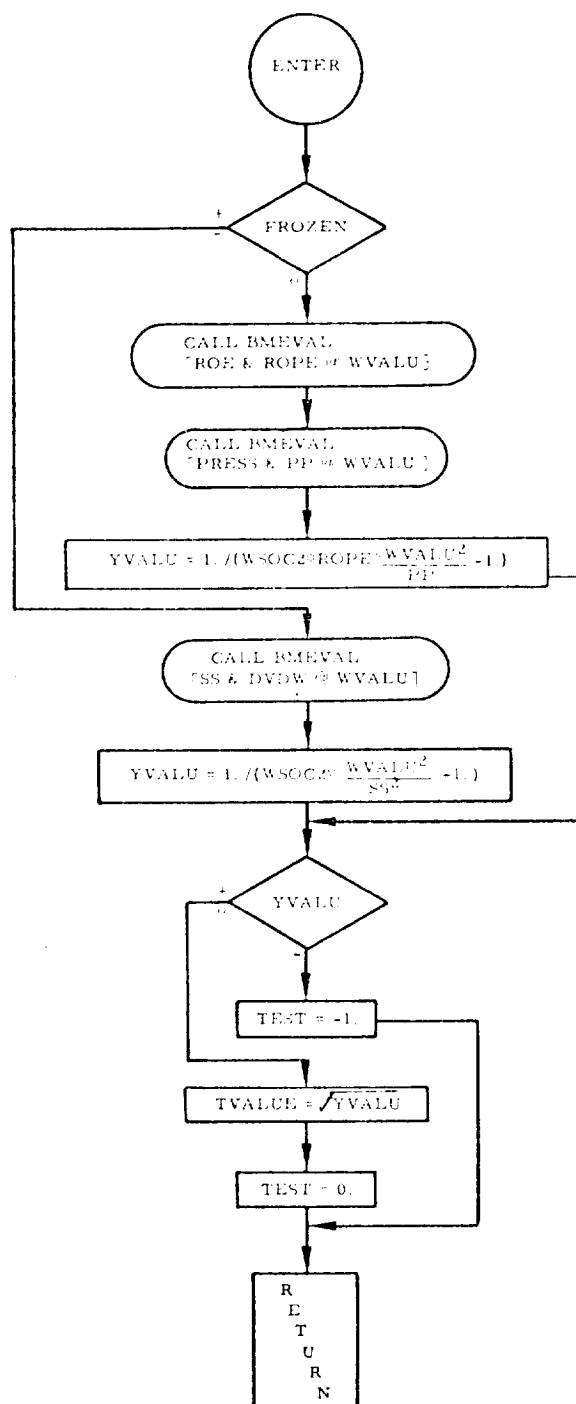
$$\tan \alpha = \sqrt{\frac{1}{[V_{\text{sonic}} W/c]^2 - 1}},$$

where the local frozen sound speed (c) is also determined from a beam fit evaluation of the gas properties table.

The following is an explanation of the subroutine call list.

- WVALU - The known value of velocity ratio
- TVALU - Value of $\tan \alpha$ corresponding to WVALU
- TEST - A signal that the input velocity ratio is subsonic. If
TEST = -1, subsonic; TEST = 0, supersonic.

Subroutine TAGAL



11. SONICP Subroutine

For an ideal gas, the SONICP Subroutine is called to adjust the units, determine the sonic values, and "beam fit" gas properties. Corresponding values of specific impulse, $\frac{\text{lb f sec}}{\text{lbm}}$; density, lbm/ft^3 ; pressure, (lb f/in^2) ; and local frozen sound speed, (ft/sec) , must be stored into variables $W(I)$, $RO(I)$, $P(I)$, and $VS(I)$, respectively. The subroutine converts the units of pressure to lb f/ft^2 and density to $\frac{\text{lb f sec}^2}{\text{ft}^4}$.

If the program is to calculate local sound speeds, the pressure and density is beam fit as a function of specific impulse to calculate the sonic velocity at the throat. The sonic I_s is first bracketed by two values of specific impulse and a halving process is used until

$$\frac{dP/dI_s}{d\rho/dI_s} = V_{\text{sonic}}^2 \pm 0.00001,$$

where:

V_{sonic}/g_0 is the value of I_s at which dP/dI_s and $d\rho/dI_s$ are evaluated. The velocity and density at this point are stored into variables SONICV and RHOSON, respectively. All of the specific impulse values are converted to velocity ratios by dividing each one by the sonic I_s . The pressure and density is then beam fit again as a function of velocity ratio.

For the option where the local frozen sound speeds are used, the curve of c vs W is beam fit for the purpose of evaluating the local speed of sound throughout the flow field. Also, the iteration to determine the sonic velocity at the throat is eliminated because this value is given in the input.

12. REALV Subroutine

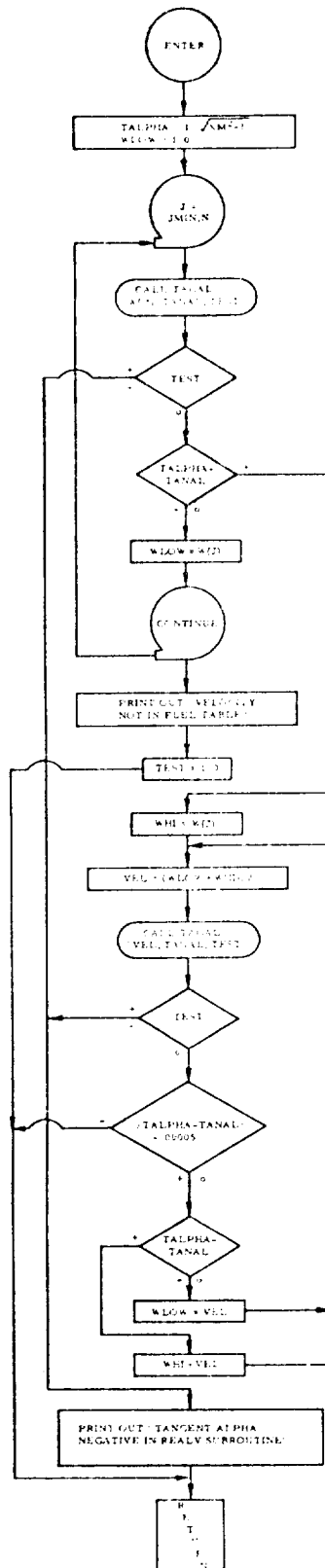
For an ideal gas, the REALV Subroutine calculates the velocity ratio (W) for the Mach number given in the call list. The corresponding value of $\tan \alpha$ is calculated by

$$\tan \alpha = 1.0/\sqrt{M^2 - 1}$$

An iteration is necessary to determine the velocity ratio. This is accomplished by using the TAGAL Subroutine for increasing values of W to calculate the corresponding values of $(\tan \alpha)_G$ until $\tan \alpha$ is bracketed. Knowing the bracketed values of W, a halving process is used; for each guess on W, the corresponding value of $(\tan \alpha)_G$ is calculated until

$$(\tan \alpha)_G = \tan \alpha \pm 0.00005$$

Subroutine REALV

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13. MFLOW Subroutine

The MFLOW Subroutine calculates the mass flow between two points along a down Mach line. After the calculation of the first five Mach lines from the exit, this subroutine is called at each interior intersection in the remainder of the flow field construction to determine when the accumulated mass flow along a down Mach line has exceeded the total mass flow through the nozzle.

To calculate the mass flow for axisymmetric flow,

$$\dot{m} = \int \left(\rho W \tan \alpha \frac{\sqrt{1 + \tan^2 \theta}}{\tan \theta - \tan \alpha} 2\pi Y \right) dY. \quad (13.1)$$

For two-dimensional flow, the $2\pi Y$ term is eliminated.

The known conditions at points 1 and 2 must be stored in variables B(I) and C(I), respectively.

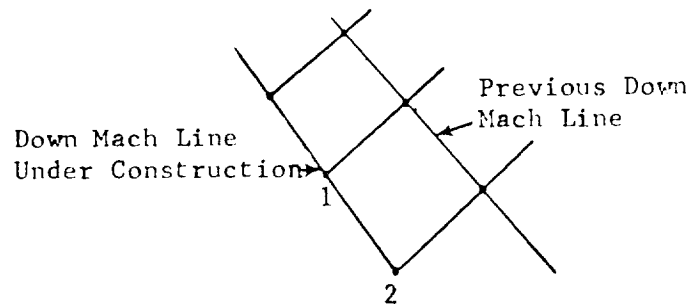


Figure 10

The increment of mass flow between the first two points on the Mach line, which is indicated by the value in N4, is calculated by trapezoidal integration. In equation (13.1), let Q represent the quantity in parenthesis; then the first mass flow increment is found by

$$\dot{m}_2 = \left(\frac{Q_1 + Q_2}{2} \right) (X_2 - X_1). \quad (13.2)$$

The remaining increments of mass flow, as illustrated in figure 11, are determined by the parabolic integration of equation (13.1).

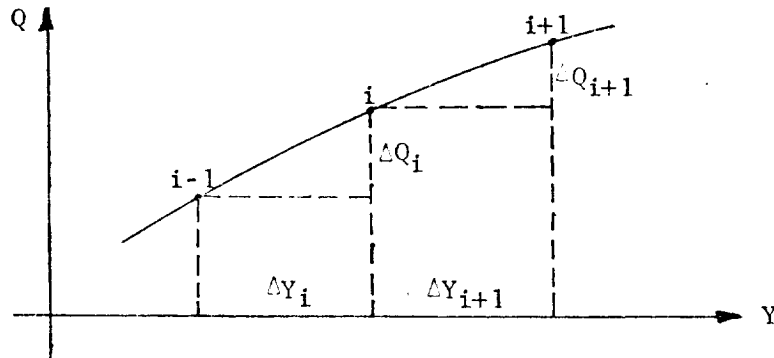


Figure 11

$$Q_{i+1} = Q_i + \Delta Y_{i+1} \left[\frac{1}{2}(Q_i + Q_{i+1}) - \frac{1}{6} \left(\frac{\Delta Y_{i+1}}{\Delta Y_i} \right) \left(\frac{\Delta Y_i \Delta Q_{i+1} - \Delta Y_{i+1} \Delta Q_i}{\Delta Y_{i+1} + \Delta Y_i} \right) \right]$$

The calculated mass flow is then added to AMFLOW; if the resultant value is greater than the total mass flow through the nozzle, a point between points 1 and 2 corresponding to the total mass flow is calculated by linear interpolation. The coordinates and flow conditions are stored in C(I) and a signal parameter, TEST, is set equal to one (1.0). If AMFLOW is less than the total mass flow, TEST is set equal to zero.

```

graph TD
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    N1 --> N2{N=2}
    N2 --> N3{N=3}
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    N4 --> N5{N=5}
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14. MFLOWT Subroutine

The MFLOWT Subroutine calculates the mass flow between two points on an up Mach line using trapezoidal integration.

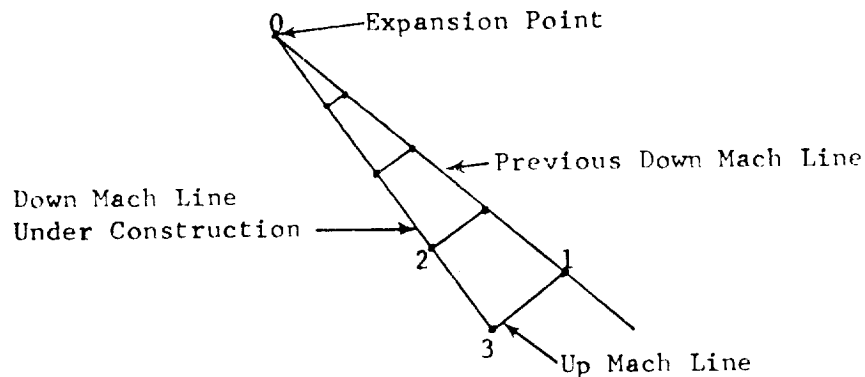


Figure 12

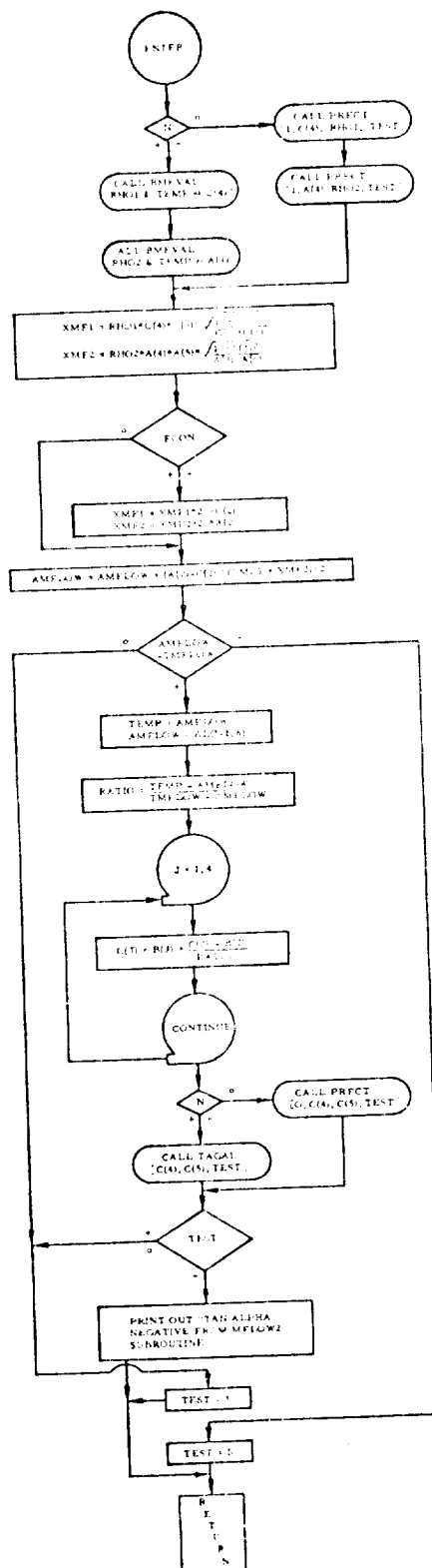
The mass flow between points 0 and 1 must be stored in variable AMFLOW, and the flow conditions at points 1 and 3 must be stored into A(I) and C(I), respectively. The integral for calculating mass flow along an up Mach line for axisymmetric flow is

$$\dot{m} = \int \rho W \tan \alpha \frac{\sqrt{1 + \tan^2 \theta}}{\tan \theta + \tan \alpha} 2\pi Y dY.$$

For two dimensional flow, the $2\pi Y$ term is omitted.

The calculated mass flow is then added to AMFLOW; if the resultant value is greater than the total mass flow through the nozzle, a point between points 2 and 3 corresponding to the total mass flow is calculated by linear interpolation. The coordinates and flow conditions are stored in C(I) and a signal parameter, TEST, is set equal to one (1.0). If AMFLOW is less than TMFLOW, TEST is set equal to zero.

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15. PERFO Subroutine

After the calculation of the points describing the nozzle contour, the PERFO Subroutine is called to calculate and print the performance parameters. The calculated contour points must be stored in CL(I,J) beginning at the throat and BL(I,J) beginning at the exit.

The vacuum gross thrust coefficient is calculated by integrating along the exit Mach line. For axisymmetric flow

$$C_{TGE} = \pi(\gamma M^2 + 1.0) \frac{P}{P_0} \frac{1 - Y_{CH}^2}{A^*}.$$

For two dimensional flow, replace Y_{CH}^2 with Y_{CH} .

The vacuum gross thrust coefficient at each point along the contour is then calculated by the following equation for axisymmetric flow.

$$C_{TG} = C_{TGE} - \int \frac{P}{P_0} 2\pi Y dY.$$

For two dimensional flow, the $2\pi Y$ term is omitted. The integration begins at the exit where trapezoidal integration is used for the first point, and parabolic integration for the remaining points. The following are the nine parameters printed at each contour point stored in CL(I,J).

PRF(1) - The ratio of the X coordinate to the throat radius as stored in CL(I,1)

PRF(2) - The ratio of the Y coordinate to the throat radius as stored in CL(I,2)

PRF(3) - The slope of the contour or $\tan \theta$ as stored in CL(I,3)

PRF(4) - The Mach number calculated by

$$M = \sqrt{1 + \frac{1}{\tan^2 \alpha}}, \text{ where } \tan \alpha = CL(I,5)$$

PRF(5) - The ratio of static pressure to chamber or total pressure,
a function of $W = CL(I,4)$

PRF(6) - The ratio of specific heats which is input for a perfect
gas, but for an ideal gas $\gamma = c^2 \rho / P$

PRF(7) - The ratio of accumulated surface area to the throat area (A^*).

For two dimensional flow, $AS/A^* = 1/A^* \int ds$

For axisymmetric flow, $AS/A^* = 2\pi/A^* \int Y ds$

where $ds = \sqrt{(dX)^2 + (dY)^2}$

PRF(8) - The vacuum gross thrust coefficient

PRF(9) - The net thrust coefficient, which is the C_{TG} less (1) friction
drag along the contour, and (2) subsonic losses. For a perfect
gas and axisymmetric flow, the frictional drag coefficient is

$$DRAG = 1/2 \int \frac{C_f}{P_o A^*} M^{2-\gamma} 2\pi Y dX.$$

Where: the coefficient of friction (C_f) at each point is determined from

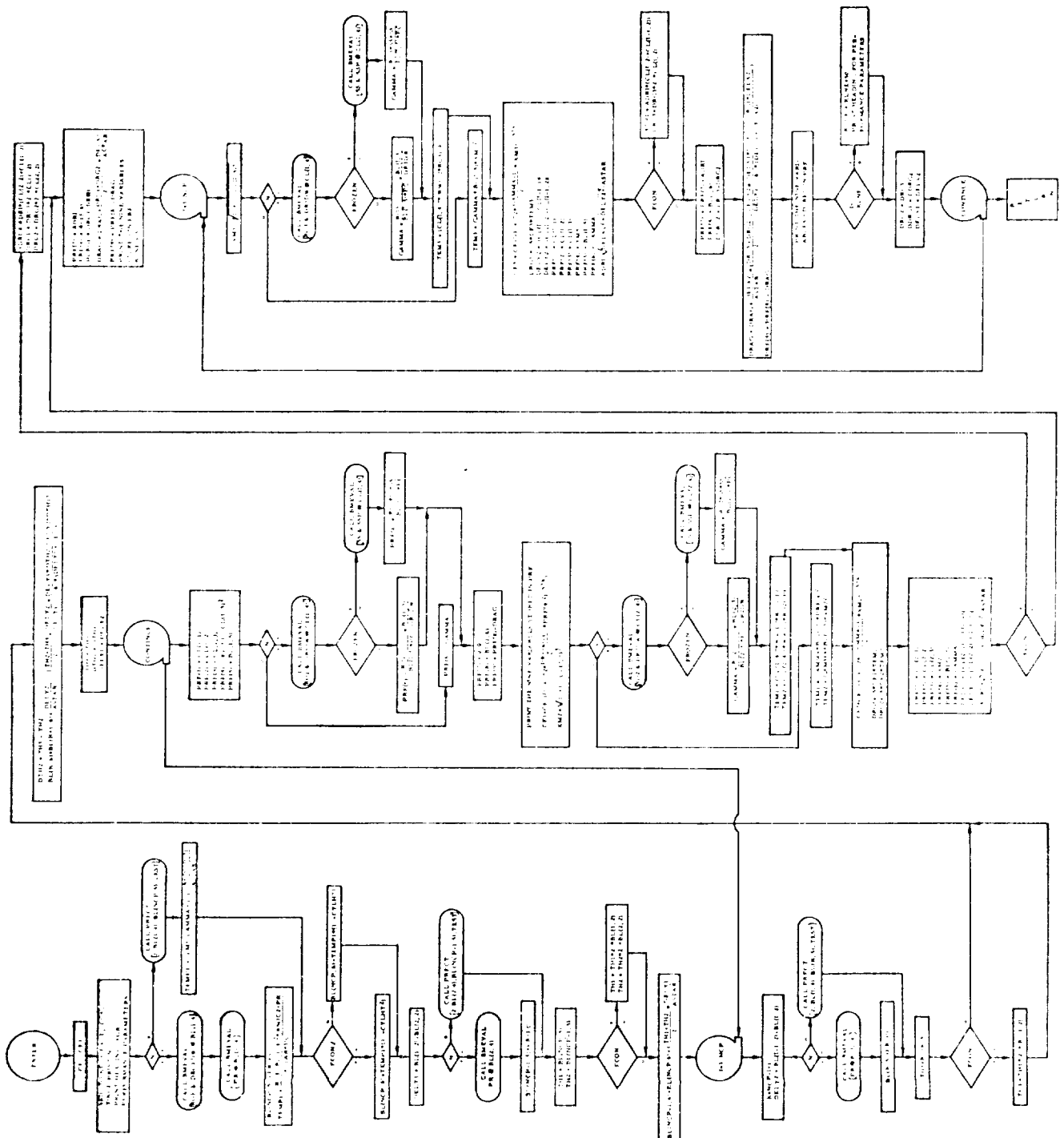
$$C_f = C_{fi} \left[1 + .72 \frac{\gamma-1}{2} M^2 \right]^{-0.578}. \quad \text{The equation for two dimensional flow}$$

is the same except the $2\pi Y$ term is omitted. For an ideal gas, $DRAG =$

$$1/2 \int \frac{C_f W^2 V_{sonic}^2 \rho}{P_o A^*} 2\pi Y dX. \quad \text{Again the } 2\pi Y \text{ term is omitted for two dimensional flow.}$$

The subsonic thrust coefficient loss is an input parameter.

Subroutine PERFO



16. BMFIT Subroutine

The BMFIT Subroutine is used to calculate sets of cubic coefficients for a spline curve fit through a series of input points. This method of curve fitting, commonly referred to as beam-fitting, is derived in Volume I.

I. The calling sequence for the subroutine is:

CALL BMFIT (L,N,X,Y,EO,EN,A,B,C,D),

where:

L - A fixed point variable denoting one of the following moment or slope end-condition options

$L = 1, M_1 = EO \text{ and } M_N = EN$

$L = 2, M_1 = EO \text{ and } M_N = M_{N-1}$

$L = 3, M_1 = EO \text{ and } Y'_N = EN$

$L = 4, M_1 = M_2 \text{ and } M_N = EN$

$L = 5, M_1 = M_2 \text{ and } M_N = M_{N-1}$

$L = 6, M_1 = M_2 \text{ and } Y'_N = EN$

$L = 7, Y'_1 = EO \text{ and } M_N = EN$

$L = 8, Y'_1 = EO \text{ and } M_N = M_{N-1}$

$L = 9, Y'_1 = EO \text{ and } Y'_N = EN$

N - A fixed point variable equal to the number of points to be fit

X - A single dimensional array containing the values of the independent variables

Y - A single dimensional array containing the values of the dependent variable

EO - The moment or slope at the leading end as required by the options

$L = 1, 2, 3, 7, 8, \text{ and } 9.$ (EO is zero for $L = 4, 5$ and $6.$)

EN - The moment or slope at the trailing end as required by the options

$L = 1, 3, 4, 6, 7, \text{ and } 9$ (EN is zero for $L = 2, 5, \text{ and } 8.$)

M_i - The moment at the i^{th} contour point

Y_i' - The slope at the i^{th} contour point.

A set of coefficients is calculated for the interval between each input point and then stored in the one dimensional arrays A, B, C, and D. For the i^{th} interval,

$$Y = A_i X^3 + B_i X^2 + C_i X + D_i.$$

```

graph TD
    ENTER((ENTER)) --> Init1[N = M  
KN = 3 * (L - 1) / 3 - 2 * L  
KO = (L - 1) / 3]
    Init1 --> J1((J =  
2, N))
    J1 --> Loop1[A(J) = X(J) - X(J - 1)  
D(J) = (Y(J) - Y(J - 1)) / A(J)]
    Loop1 --> CONT1((CONTINUE))
    CONT1 --> K2[K2 = N - 2]
    K2 --> KN1{KN - 1}
    KN1 -- + --> Init2[C(N) = 3 * (SLOPEN - D(N)) / A(N)  
B(N) = 0.5  
K3 = N  
K1 = 1]
    KN1 -- - --> Init2
    Init2 --> T2[T = 2 * (A(N) + A(N - 1))  
B(N) = SLOPEN]
    T2 --> T3[T = 3 * A(N) + A(N - 1) + A(N - 1)  
SLOPEN = 0.0]
    T3 --> Init3[C(N - 1) = (6 * (D(N) - D(N - 1)) - SLOPEN * A(N)) / T  
B(N - 1) = A(N - 1) / T  
K3 = N - 1  
K1 = 2]
    Init3 --> J2((J =  
K1, K2))
    J2 --> Loop2[K = N - J  
T = 2 * (A(K) + A(K + 1)) - A(K + 1) * B(K + 1)  
B(K) = A(K) / T  
C(K) = (6 * (D(K + 1) - D(K)) - A(K + 1) * C(K + 1)) / T]
    Loop2 --> CONT2((CONTINUE))
    CONT2 --> KO1{KO - 1}
    KO1 -- + --> J1
    KO1 -- - --> J2

    J2 --> Init4[B(1) = (6 * (D(2) - SLOPEO) - C(2) * A(2)) / (A(2) * (2 - B(2)))  
K1 = 2]
    Init4 --> Init5[B(1) = SLOPEO  
B(2) = C(2) - B(2) * B(1)]
    Init5 --> Init6[B(1) = (6 * (D(3) - D(2)) - A(3) * C(3)) / (3 * A(2) + A(3) * (2 - B(3)))  
B(2) = B(1)]
    Init6 --> K13[K1 = 3]
    K13 --> J3((J =  
K1, K3))
    J3 --> Loop3[B(J) = C(J) - B(J) * B(J - 1)]
    Loop3 --> CONT3((CONTINUE))
    CONT3 --> KN1_2{KN - 1}
    KN1_2 -- + --> B_N[B(N) = B(N - 1)]
    KN1_2 -- - --> J4((J = 2, N))
    B_N --> J4
    J4 --> Init7[TA = 6 * A(J)  
TM = B(J - 1) * X(J)  
TN = X(J - 1) * B(J)  
TX = X(J) * (X(J) - X(J - 1) - X(J - 1))  
TY = X(J - 1) * (X(J) + X(J) - X(J - 1))  
A(J - 1) = (B(J) - B(J - 1)) / TA  
C(J - 1) = D(J) + (B(J) * (-TX) + 2 * X(J - 1) * TN - B(J - 1) * TY - 2 * X(J) * TM) / TA  
B(J - 1) = (TM - TN) * 3 / TA  
D(J - 1) = (TX * TN + TY * TM) / TA + Y(J - 1) - X(J - 1) * D(J)]
    Init7 --> CONT4((CONTINUE))
    CONT4 --> RETURN[RETURN]

```

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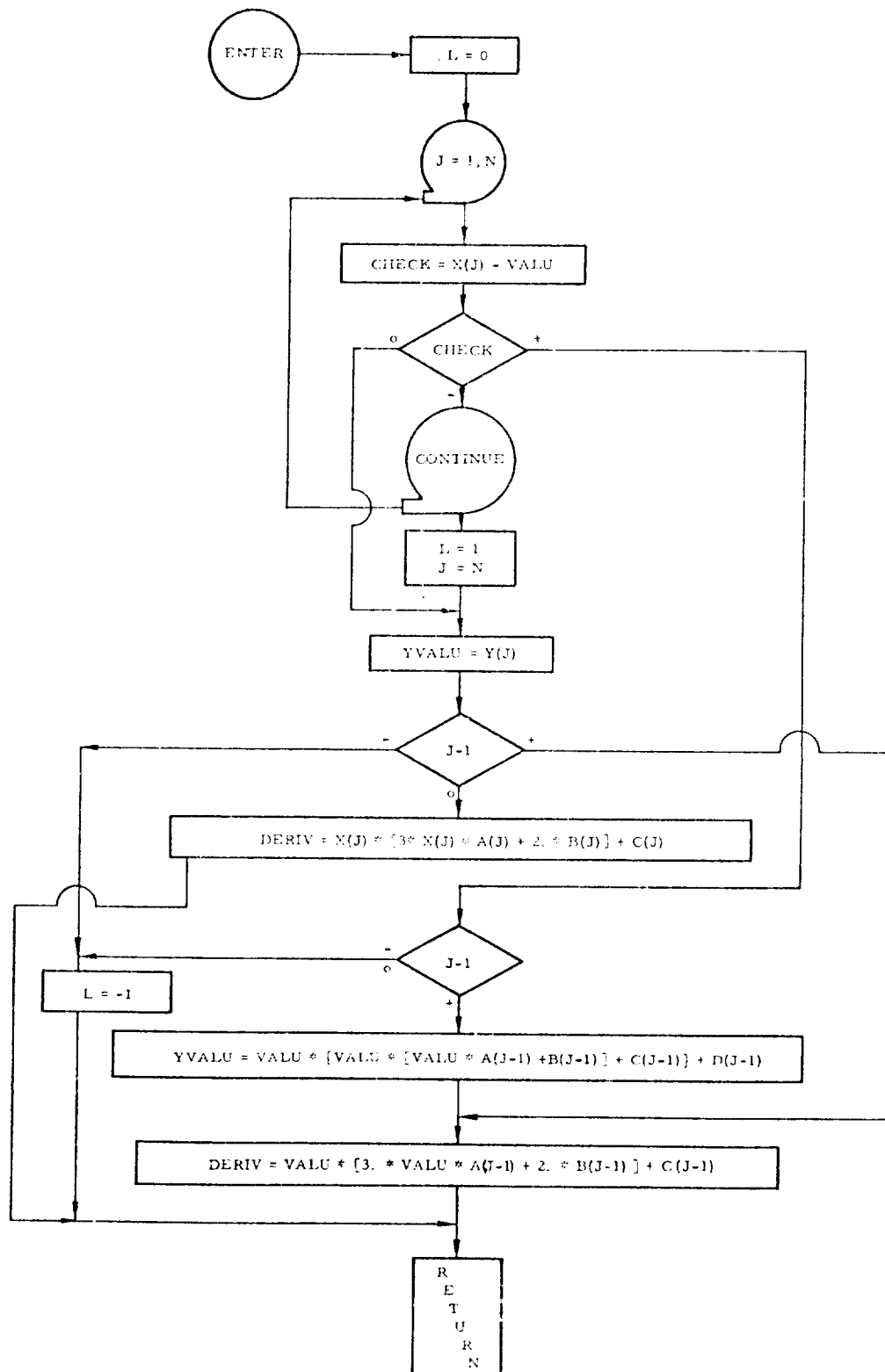
17. BMEVAL Subroutine

After a curve that is represented by a set of input coordinates has been fit by the BMFIT Subroutine, the BMEVAL Subroutine is used to evaluate the curve and its first derivative for a given value of the independent variable. The subroutine solves the cubic equation $Y = AX^3 + BX^2 + CX + D$ after searching for the set of coefficients corresponding to the given value of the independent variable.

The eleven parameters that make up the call list of the BMEVAL Subroutine are as follows:

- N - The number of coordinates describing the curve
- X - Contains values of the independent variable in increasing order (maximum of 100)
- Y - Corresponding values of the dependent variable (maximum of 100)
- VALU - Value of the independent variable at which the curve is to be evaluated.
- L - Error signal
 - L = -1, curve is out of range to the left
 - L = 0, curve is in range
 - L = +1, curve is out of range to the right
- A,B, - Contain the coefficients of the cubic equation for intervals
C,D between each input coordinate (these coefficients are calculated in the BMFIT Subroutine)
- YVALU - The calculated value of the dependent variable corresponding to VALU
- DERIV - Calculated value of the first derivative corresponding to VALU.

Subroutine BMEVAL



SECTION III INPUT - OUTPUT

The INPUT Subroutine is used to initialize constants and to load the input data. After loading the required data, this subroutine reads any of the optional input with a scatter loading feature that terminates with an END card. The following is a detailed outline of the input procedure. A sample input sheet is shown in figure 13, which corresponds to the output results given in Paragraph B.

A. INPUT FORMAT

All data cards must contain the FORTRAN variable name in card columns 1 - 6 (adjusted to the left). For single-valued variables, the number must be in columns 7 - 16. Depending on the gas model option, the parameters under Required Input must be in the order listed. The Optional Input has no particular order and is terminated by a card with END in columns 1 - 3. Each optional input variable will equal the built-in value, unless another number is input. The built-in value is restored between multiple cases.

1. Required Input

a. General

TITLE — Any information to be printed as a heading at the beginning of the output will be placed in columns 7-72.

EM — Design or exit Mach number

b. Gas Model Option

(1) Perfect Gas (Constant Specific Heat Ratio)

GAMMA — Specific heat ratio

(2) Ideal Gas (Table of gas properties is required and must follow the END card)

(a) Local Sound Speeds Calculated by Program

XN — The number of cards in the table of gas properties

PC — Chamber pressure, psi

(b) Local Frozen Sound Speeds from Table of Gas Properties

FROZEN Must be non-zero

XN — The number of cards in the table of gas properties

PC — Chamber pressure, psi

2. Optional Input

Name	Built-In Value	Description
FCON	1.0	FCON = 1.0, axisymmetric flow FCON = 0.0, two-dimensional flow
FM	1.005	Throat Mach number or Mach number at which calculations end
CYLHT	.01	Ratio of the plug height to throat radius at the exit
XNUM	60.	Number of points on the exit Mach line
WEXPAN	.0001	Initial increment of velocity for corner expansion
EXPMAX	.005	Maximum expansion increment
DELTB	-.001	Increment of $\tan \theta$ for bracketing contour point in PLUGPT Subroutine
FPRINT	0.0	FPRINT = 0.0, no flow field print FPRINT = 1.0, flow field print
CFI	.003	Incompressible skin friction coefficient along nozzle contour
DRAG	.011	Thrust loss at the throat

Name	Built-In Value	Description
TOLMF	.000001	Tolerance on mass flow balance
XTOL	.000005	Iteration tolerance for Mach line intersections
PHI	-.5	Angle of rotation for interior point intersections (radians)

A card with END in columns 1 - 3 must follow the last optional input card.

For an ideal gas, a table of gas properties must be input. The first card is a title card describing the gas model, and the second card contains in column 2 - 15 the specific impulse, $\left(\frac{\text{lb-f-sec}}{\text{lbm}}\right)$, at the throat. Each of the remaining cards must contain corresponding values of specific impulse $\left(\frac{\text{lb-f-sec}}{\text{lbm}}\right)$, pressure (psi), and density (lbm/ft^3) in columns 2-15, 16-29, and 30-43, respectively, with the local sound speed (ft/sec) in columns 58 - 71.

B. OUTPUT DESCRIPTION

The first page of the output includes the title and values of important input parameters. If there is no flow field printout, the following lines of output beginning on the next page are the nine performance parameters at points along the contour, as explained in the PERFO Subroutine. A sample of the program output is shown in figures 14a and 14b.

C. PROCEDURES FOR CORRECTING PROGRAM FAILURES

If the flow field construction is close to the throat, it is normal for the calculations to end due to a subsonic velocity before the desired minimum velocity is reached. Even though a program failure is indicated, the performance is printed beginning with the point of the smallest supersonic velocity calculated on the contour.

SINGLE EXPANSION PLUG NOZZLE DESIGN

SAMPLE TEST CASE

*** INPUT ***

CONSTANT GAMMA,
AXISYMMETRIC FLOW

EM = 2.00000E 00	PC = 0.	GAMMA = 1.40000E 00	FM = 1.00500E 00	CYLPT = 2.00000E-02
XNUM = 6.00000E 01	PCON = 1.00000E 00	WEXPAN = 10.00000E-05	EXP MAX = 5.00000E-03	DELTA = -1.00000E-03
FPRINT = 0.	CF1 = 3.00000E-03	DRAG = 1.10000E-02		

SONIC VELOCITY = 0.40425E+00
 TANGENT OF ALPHA BECAME NEGATIVE IN INT1 SUBROUTINE
 VELOCITY BECAME SUPERSONIC ALONG MACH LINE

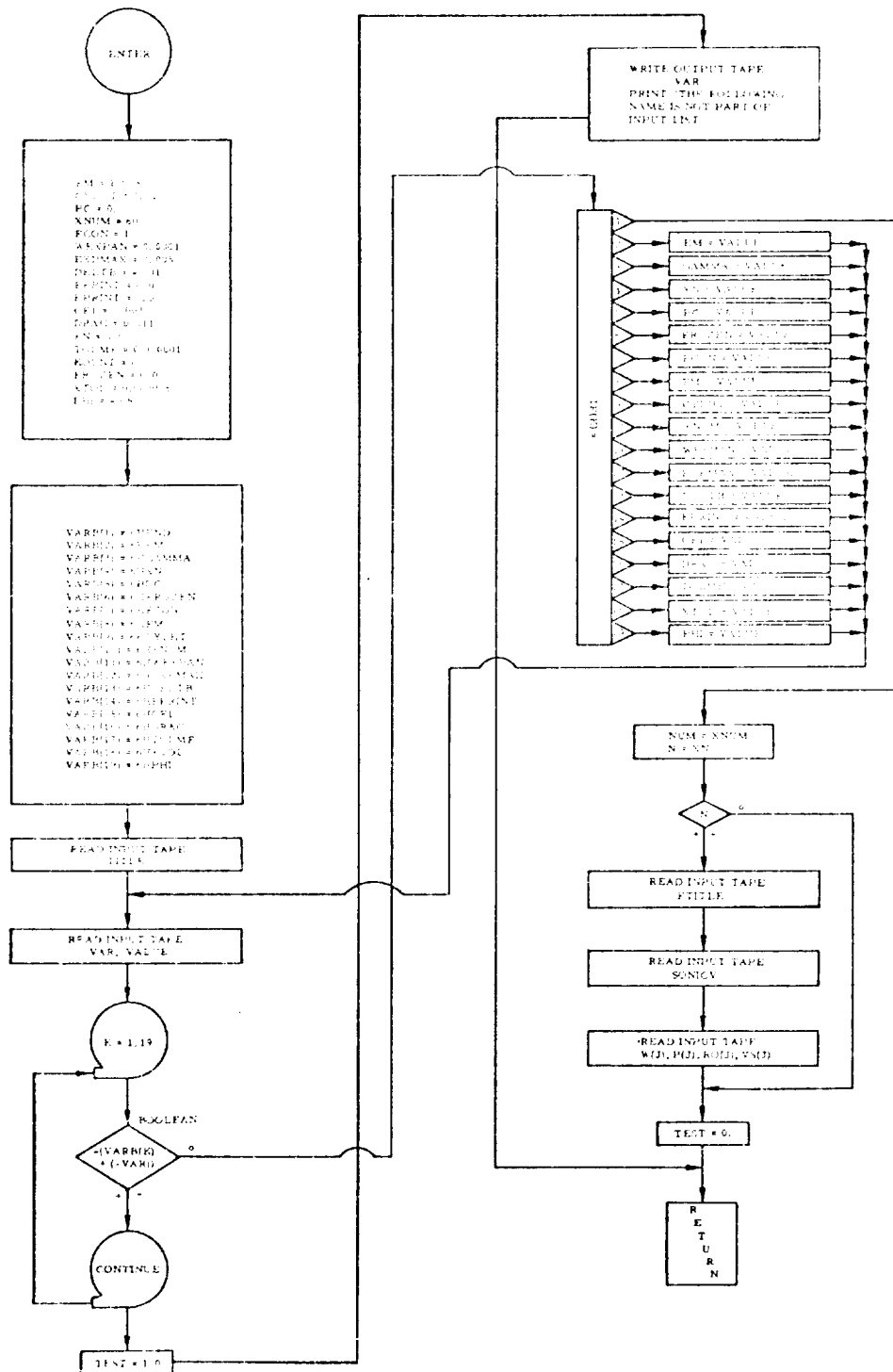
X/R	Y/R	TAN THETA	MACH NO.	P/PC	GAMMA	AS/A*	CTG	CTN
-0.0591261	0.6055172	-0.7341519	1.0207583	0.5155558	1.4000000	0.	1.1995926	1.1885926
-0.0415766	0.5956042	-0.7234208	1.0390886	0.5044482	1.4000000	0.0442996	1.2129806	1.2019432
-0.0251006	0.5837521	-0.7131692	1.0566783	0.4933754	1.4000000	0.0846994	1.2247609	1.2136889
-0.0092799	0.5726654	-0.7026706	1.0737632	0.4837253	1.4000000	0.1224140	1.2353407	1.2242358
0.0062695	0.5618150	-0.6913257	1.0906158	0.4738213	1.4000000	0.1587278	1.2452904	1.2341534
0.0215297	0.5511447	-0.6811571	1.1075288	0.4639933	1.4000000	0.1937137	1.2546913	1.2435232
0.0466800	0.5408527	-0.6702140	1.1237855	0.4546554	1.4000000	0.2274780	1.2634064	1.2522078
0.0524096	0.5305072	-0.6596260	1.1403237	0.4452680	1.4000000	0.2615285	1.2718253	1.2605955
0.0680693	0.5200923	-0.6479386	1.1569271	0.4359604	1.4000000	0.2948790	1.2799602	1.2686997
0.0840938	0.5098872	-0.6362765	1.1731626	0.4269745	1.4000000	0.3279180	1.2876205	1.2763291
0.1005234	0.4995037	-0.6247484	1.1895756	0.4180087	1.4000000	0.3610375	1.2950971	1.2837746
0.1176172	0.4889982	-0.6127142	1.2070144	0.4086153	1.4000000	0.3945190	1.3023425	1.2909881
0.1351359	0.4786153	-0.5999602	1.2240822	0.3995561	1.4000000	0.4277849	1.3091962	1.2978096
0.1530650	0.4679424	-0.5879178	1.2411334	0.3906404	1.4000000	0.4611268	1.3159346	1.3045158
0.1715860	0.4573856	-0.5758856	1.2583331	0.3817848	1.4000000	0.4946221	1.3224566	1.3110052
0.1908256	0.4463421	-0.5630516	1.2755121	0.3730795	1.4000000	0.5282617	1.3286662	1.3171818
0.2107510	0.4352397	-0.5506378	1.2927999	0.3644611	1.4000000	0.5622087	1.3347595	1.3232416
0.2314736	0.4238865	-0.5384157	1.3103160	0.3558751	1.4000000	0.5964768	1.3406902	1.3291384
0.2531004	0.4124487	-0.5256463	1.3278673	0.3474202	1.4000000	0.6310212	1.3463689	1.3347826
0.2757004	0.4004976	-0.5123644	1.3454636	0.3390934	1.4000000	0.6658704	1.3518131	1.3401917
0.2993207	0.3888313	-0.4994689	1.3634051	0.3307579	1.4000000	0.7012320	1.3572010	1.3455442
0.3240955	0.3765603	-0.4873061	1.3814865	0.3225156	1.4000000	0.7369553	1.3623801	1.3506873
0.3501226	0.3640746	-0.4747335	1.3996974	0.3143748	1.4000000	0.7730483	1.3673514	1.3556221
0.3775104	0.3513359	-0.4611189	1.4180594	0.3063292	1.4000000	0.8095285	1.3721262	1.3603599
0.4063814	0.3383042	-0.4477249	1.4365597	0.2983710	1.4000000	0.8464067	1.3767135	1.3649098
0.4369756	0.3249383	-0.4342217	1.4553518	0.2904903	1.4000000	0.8836858	1.3811199	1.3692782
0.4691531	0.3111827	-0.4207253	1.4743617	0.2826733	1.4000000	0.9213644	1.3853533	1.3734733
0.5033920	0.2970164	-0.4072309	1.4936547	0.2749159	1.4000000	0.9594094	1.3894086	1.3774900
0.5397877	0.2824477	-0.3936279	1.5132316	0.2672242	1.4000000	0.9977594	1.3932720	1.3813144
0.5785808	0.2673888	-0.3801155	1.5331688	0.2595756	1.4000000	1.0363858	1.3969540	1.3849573
0.6200231	0.2518671	-0.3664937	1.5534498	0.2519245	1.4000000	1.0751783	1.4004344	1.3883986
0.6643980	0.2359111	-0.3525469	1.5740623	0.2444627	1.4000000	1.1140096	1.4036961	1.3916212
0.7120141	0.2195903	-0.3379216	1.5949817	0.2370260	1.4000000	1.1527158	1.4067179	1.3946040
0.7632761	0.2028279	-0.3228311	1.6163709	0.2296247	1.4000000	1.1911776	1.4095072	1.3973550
0.8188491	0.1851553	-0.3086865	1.6385922	0.2221491	1.4000000	1.2293720	1.4121225	1.3999323
0.8789332	0.1674451	-0.2922062	1.6610427	0.2148140	1.4000000	1.2666584	1.4144262	1.4021991
0.9447142	0.1485739	-0.2774447	1.6848201	0.2072796	1.4000000	1.3031678	1.4165516	1.4042888
1.0166969	0.1293105	-0.2615249	1.7096139	0.1996749	1.4000000	1.3381244	1.4183912	1.4060944
1.0960099	0.1095468	-0.2445506	1.7358284	0.1919080	1.4000000	1.3710840	1.4199525	1.4076239
1.1841462	0.0891463	-0.2272890	1.7644647	0.1837372	1.4000000	1.4014291	1.4212389	1.4088815
1.2826354	0.0684069	-0.2086666	1.7968590	0.1748780	1.4000000	1.4282516	1.4222297	1.4098471
1.3940609	0.0481734	-0.1878733	1.8384840	0.1640728	1.4000000	1.4605009	1.4229058	1.4105028
1.5209005	0.0294339	-0.1519865	1.9084826	0.1540015	1.4000000	1.4672991	1.4232982	1.4108801
1.6088966	0.0214279	-0.1177650	1.9466935	0.1472961	1.4000000	1.4748860	1.4234089	1.4109842
1.6542183	0.0206698	-0.08317656	1.9466935	0.1388355	1.4000000	1.4781074	1.4234166	1.4109891
1.6772986	0.0201403	-0.0414211	1.9736531	0.1331471	1.4000000	1.4796979	1.4234216	1.4109928
1.6887967	0.0200251	-0.0059102	1.9862696	0.1301572	1.4000000	1.4804776	1.4234227	1.4109931
1.6945388	0.0200026	-0.0019492	1.9960292	0.1285963	1.4000000	1.4808656	1.4234229	1.4109930
1.6974098	0.0199998	0.	2.0000000	0.1278045	1.4000000	1.4810594	1.4234229	1.4109929

Figure 14b

Whenever any failure occurs, the performance of any portion of the contour which has been calculated will be printed. If the smallest Mach number is not reasonably close to Mach 1, usually one of the following procedures will allow the flow field to be constructed closer to the throat.

1. If the mesh is too large, increase the number of points on the exit Mach line with a larger value for XNUM and/or decrease the expansion increment with a smaller value for EXPMAX.
2. For an iteration failure of the mass flow balance in the PLUGPT Subroutine, adjust cylinder height and/or the iteration tolerance.
3. For high exit Mach number designs the flow is turned more than 90° at the expansion point. In this case, the program ends calculations when the Y/R_t coordinate on the contour is greater than 1.0.

Subroutine INPUT



APPENDIX A
SYMBOL TABLE

A^*	-	Theoretical throat area
c	-	Local speed of sound
C_f	-	Coefficient of friction
C_{TG}	-	Gross thrust coefficient
C_{TN}	-	Net thrust coefficient
I_s	-	Specific impulse
M	-	Mach number
P	-	Pressure
V_{max}	-	Maximum velocity
V_{sonic}	-	Sonic velocity
W	-	Velocity ratio; either V/V_{max} or V/V_{sonic}
\dot{x}	-	Mass flow rate
α	-	Mach angle
γ	-	Ratio of specific heats
ρ	-	Density
σ	-	=1 for axisymmetric flow =0 for two-dimensional flow
θ	-	Angle between velocity vector and axis of symmetry

APPENDIX B
FORTRAN SYMBOL TABLE
(COMMON)

Variable	Dimension	Description
A1	100	Coefficients of X^3 for c vs W beam fit
A2	100	Coefficients of X^3 for ρ vs W beam fit
A3	100	Coefficients of X^3 for P vs W beam fit
AL	200,6	Variable used to store, X, Y, tan θ , W, tan α , and \dot{w} on a down Mach line
AMFLOW	-	Accumulated mass flow
A	5	For storing X, Y, tan θ , W, and tan α ; usually represents the point on an up Mach line
ASTAR	-	Minimum throat area
B1	100	Coefficients of X^2 for c vs W beam fit
B2	100	Coefficients of X^2 for ρ vs W beam fit
B3	100	Coefficients of X^2 for P vs W beam fit
BL	500,6	Variable used to store, X, Y, tan θ , W, tan α and \dot{w} on a down Mach line
B	5	For storing X, Y, tan θ , W, and tan α ; usually represents the point on a down Mach line
C1	100	Coefficients of X for c vs W beam fit
C2	100	Coefficients of X for ρ vs W beam fit
C3	100	Coefficients of X for P vs W beam fit
CF1	-	Incompressible coefficient of skin friction
CL	500,6	Variable used to store, X, Y, tan θ , W, tan α , and \dot{w} at calculated points along the contour
C	5	For storing X, Y, tan θ , W, and tan α ; normally used in the calculation of a Mach line intersection

Variable	Dimension	Description
CYLHT	-	Value of Y/R at the exit contour point
D1	100	Contains the constants from the beam fit of sound speeds
D2	100	Contains the constants from the beam fit of densities
D3	100	Contains the constants from the beam fit of pressures
DELTA	-	Increment of $\tan \theta$ for bracketing a contour point in mass flow balance iteration
DRAG	-	Value of accumulated drag along contour
EM	-	Input value of exit Mach number
EXPMAX	-	Maximum increment of W for corner expansion
FCON	-	Indicates two-dimensional or axisymmetric flow
FM	-	Input Mach number for constant Mach number starting line
FPRINT	-	Indicates whether flow field is to be printed
FROZEN	-	For an ideal gas this indicates that input values of local frozen sound speeds are used
GAMMA	-	Specific heat ratio when constant
JMIN	-	The number of the first supersonic value of W in table of gas properties
NCP	-	The number of calculated contour points
N	-	Number of cards making up table of gas properties
NUM	-	The number of points along exit Mach line
PC	-	Chamber pressure, psi, for ideal gas option
P	100	Contains the values of pressure for ideal gas
RHOSON	-	Density evaluated at the sonic velocity
RO	100	Contains the values of density for ideal gas

Variable	Dimension	Description
SONICV	-	The sonic velocity
TITLE	10	To store and print the title card
TMFLOW	-	Total mass flow through the nozzle
TOLMF	-	Iteration tolerance on mass flow balance
VS	100	Contains the values of local sound speeds for ideal gas
WEXPAN	-	Starting increment of W for corner expansion
W	100	Contains the values of velocity ratio for ideal gas
WSOC2	-	The sonic velocity squared
XN	-	Floating point variable for N
XNUM	-	Floating point variable for NUM
XTOL	-	Iteration tolerance for Mach line intersections.